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OBTAINING THE OPTIMAL SHORT-TERM HYDROTHERMAL COORDINATION SCHEDULING: A STOCHASTIC VIEW POINT

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Abstract

In this document we develop a non-linear, stochastic and integer model for the problem of determining the optimal hourly schedule of power generation in a hydrothermal power system. We consider a power generation system comprising thermal and hydro units and the problem concerns the scheduling of operation levels for all power units and considering the hydro constrains, such that the operation costs over the time horizon are minimal. The level of customer service equation is introduced and the power balance constraint, total water discharge constraint, reservoir volume limits and constraints on the operation limits of the hydrothermal generator and the thermal generator are fully accounted for. The proposed problem is illustrated and tested on two model systems using a random search technique and genetic algorithm. We report the practical and theoretical results.

1. Introduction

The systematic coordination of the operation of a system formed by hydroelectric generation plants is a classical problem involving the planning

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of the operation of a hydraulic generation system and a thermal system. The generation scheduling problem consists of determining the optimal operation strategy for the next scheduling period, subject to a variety of constrains. In literature this is known as the hydrothermal generation scheduling problem (HGSP) [9]. Most versions involve the allocation of generation among the hydro-electric and thermal plants so as to minimize the total operation costs of thermal plants while satisfying the various constrains on the hydraulic and power systems network. Usually, the short term period covers from 1 to 7 days, and then, this period is subdivided into smaller time intervals of 1 to 4 hours in which the information of the system is known and the decision variables should be optimized.

This is one of the most important problems associated with the management of a power utility and can be viewed as a problem of production planning, where the good produced is electricity and it is generated from two sources, a set of hydroelectric generating plants and a set of thermal power plants. Here, the problem of inventories does not exist because, the good produced must be delivered to the customer at time that it is generated. The master programming scheduling (MPS) is to develop the programming of system operation for each period specifying the state and the generation level of the thermal set, subject to fundamental constrains that must be satisfied such that the covering of each hourly load (demand), satisfaction of spinning, reserve requirements and transmission capacity limits, the limited energy storage capability of water reservoirs and other. Under some assumptions (such as determinism, for example), the mathematical model can be written in terms of a nonlinear objective function subject to a set of linear or nonlinear constrains. In the stochastic approach, the model includes some parameters as random variables, where the most representative is the required demand. To model the problem more realistically, the load demand, the water inflow rate and the reservoir levels of the hydroelectric plants are considered random and therefore, the mathematical complexity increases significantly. Anyway, an efficient generation schedule not only reduces the production cost but also increases the system reliability securing valuable reserves, regulating margins, and maximizing the energy capability of the reservoirs [34]. In this paper a new model is proposed using the framework stated in [12]. We addresses the hydrothermal generation scheduling problem from a stochastic view point. In our approach, we include the level of customer service, and the variable costs of operation, the demand and the spillage are considered as random variables. In our approach we use a random search method and a genetic algorithm to solve the problem. Comparative aspects of accuracy and speed of convergence are reported.

The rest of the paper is organized as follows: Section 2 presents and discusses related work, Section 3 presents the proposed mathematical model. In Section 4 a numerical example is developed. Section 5 proposes a scheme using a random search method and genetic algorithms, and Section 6 presents our conclusions.

2. Literature Review

The solution methods of the HGSP problem have been approached from several perspectives, however, literature comprises them in five major areas: (a) Lagrangian relaxation, (b) Metaheuristic decomposition, (c) Bender's decomposition, (c) Dynamic programming, and (d) Mixed integer programming.

The Lagrangian relaxation technique uses the Lagrange multipliers to relax system wide demand and reserve requirements. This method decomposes the main problem into unit-wise subproblems that are much easier to solve. Then, the Lagrange multipliers are updated at the high level, typically using a subgradient method [18]. There are many variants of this technique [2, 19, 25, 26, 28-31, 33] but all of them are underpinned by the idea of forming an objective function penalized with model constrains forming the Lagrangian function.

Metaheuristics are a class of approximate methods that have been developed strongly since their inception in the early 1980's. They are designed to optimize complex problems where classical heuristics and optimization methods have failed to be effective and efficient. Metaheuristics include, but are not limited to: constraint logic programming, genetic algorithms, greedy random adaptive search procedures, neural networks, non-monotonic search strategies, problem and heuristic space-search, simulated annealing, tabu search, threshold algorithms, and others [23]. In connection with the HGSP, there is an important class of techniques called the *heuristic decomposition methods*. These techniques decompose the HGSP problem into hydro and thermal subproblems. The hydro optimization subproblems use either the thermal cost functions or the thermal system marginal cost to efficiently allocate the water resources within the scheduling horizon [5, 7, 34]. Then, the hydro generation and reserve contributions are subtracted from the load and reserve requirements; the thermal subproblems solves a standard unit commitment problem.

Benders decomposition is used to solve the multiperiod HGSP problem and is a natural way to decompose it because the $0/1$ variable decisions are decoupled from continuous variables 17. In general, the method fixes the start-up and shut-down schedules of the thermal units, while the Benders subproblem solves a multiperiod optimal power flow. Then, the subproblem sends to the master problem marginal information on the goodness of the proposed start-up and shut-down schedule, which allows the master problem to suggest an improved start-up and shut-down schedule and, so on [3, 16].

In the general approach of dynamic programming, the problem is decomposed into a thermal subproblem and a hydro subproblem. The algorithm obtains the non discrete states to substitute the discrete states of water volume levels at each time period and then determines an optimal generation schedule while achieving the minimum fuel cost of the power system. The spinning reserve of all units provided can satisfy the requirements of the system for any unexpected change in load or loss of maximum on line generation unit, [6, 10, 27, 32].

This paper proposes the use of random coefficients with minimum variance cost (due to the use of short periods of planning) in the objective function, demand as a random variable normally distributed, and water inflow to the dam and spillage are also random variables. An important consideration, is the use of a reliability function associated to the power balance equation (customer service level), and the variable and fixed costs of each production unit. Thus, this model can be characterized as a stochastic, non-linear and integer problem. For a more completed survey of literature on the various optimization methods applied to solve the problem see Farhat and El-Hawary [22].

3. The Mathematical Model

In the construction of our proposal we use some ideas developed in [12], i.e., we also consider the scheduling of start-up/shutdown decisions and operation levels for all power units as a stochastic process. Let the planning horizon be discretized into $t \in T$ uniform subintervals, we define the sets $\mathcal{S}$ and $\mathcal{H}$, of thermal and hydro units respectively, each with its own reservoir and turbine of power generation, as shown in Figure 1. Thus, for all $i \in \mathcal{S}$ and $j \in \mathcal{H}$, the notation used is[1]

t	Time interval index (hour).
p_{it}	Power output of i-th thermal unit in megawatts at time t.
$p_i^{\min}$	Minimum power output of i-th thermal unit in megawatts.
$p_i^{\max}$	Maximum power output of i-th thermal unit in megawatts.
g_i	Fixed operating costs of i-th thermal unit in \$/h.
p_{jt}	Power output of j-th hydro plant in megawatts.
$p_{jt}^{\min}$	Minimum power output of j-th hydro plant in megawatts.
$p_{jt}^{\max}$	Maximum power output of j-th hydro plant in megawatts.
q_{jt}	Water discharge rate of j-th hydro plant during interval t, in m^3/h.
$q_{jt}^{\min}$	Lower bound for water discharge, during interval t, in m^3/h.
$q_{jt}^{\max}$	Upper bound for water discharge, during interval t, in m^3/h.
D_t	Random energy demand in megawatts.
k_{jt}	Fixed operating costs of j-th hydro plant in \$/h.
W_j	Capacity of the j-th reservoir in m^3.
$w_{j,t}$	Storage volume of j-th reservoir at end of t in m^3/h.

[1] The standard measurement unit of water flow quantities is m^3/s, however, in this document the water flow quantities are expressed in m^3/h to avoid the use of conversion coefficients in equations.

$w_{j,t}^{\min}$ Lower bound of storage volume of j-th reservoir at end of t, in m^3.

$S_{j,t}$ Spillage rate over j-th reservoir during t, in m^3/h.

$r_{j,t}$ Random water inflow rate of j-th reservoir during t, in m^3/h.

In this formulation we assume that the cost of thermal units at time t is given by the function $a_{it} + b_{it}p_{it} + c_{it}p_{it}^2$, where the coefficients a, b and c are considered as random variables, for all $i \in \mathcal{S}$ and $t \in \mathcal{T}$. Then, for each $t \in T$, the mathematical model is to minimize the function

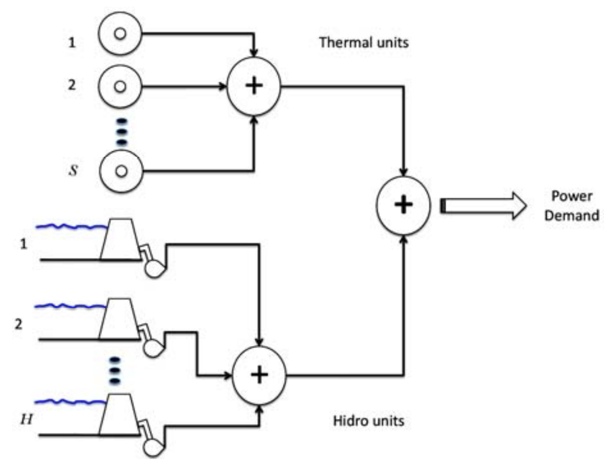


Figure 1. The system considered.

$$g(p) = \mathbf{E}\left[\sum_{i \in \mathcal{S}} y_{it}(a_{it} + b_{it}p_{it} + c_{it}p_{it}^2 + g_{it}) + \sum_{j \in \mathcal{H}} k_{jt}z_{jt}\right], \tag{1}$$

Subject to

$$\mathbf{P}\left(D_t \leq \sum_{i \in \mathcal{S}} p_{it} + \sum_{j \in \mathcal{H}} p_{jt}\right) = 1 - \alpha, \text{ (Power balance)} \tag{2}$$

$\alpha \in (0, 1)$.

$$w_{jt} = w_{j,t-1} - (q_{jt} - s_{jt} + r_{jt})t \text{ (Water balance)}, \tag{3}$$

$$q_{jt} = \beta_0 + \beta_1 p_{jt} + \beta_2 p_{jt}^2 \text{ (Water use rate characteristics)}, \tag{4}$$

$$p_i^{\min} \le p_{it} \le p_i^{\max} \text{ (Operating limits of } i\text{-th thermal unit)}, \tag{5}$$

$$q_{jt}^{\min} \le q_{jt} \le q_{jt}^{\max} \text{ (Water discharge rate limits)}, \tag{6}$$

$$w_{jt} \ge w_{jt}^{\min}, \text{ (Limit of water stored in reservoir } j\text{-th at the end of } (t)), \tag{7}$$

$$y_{it} = \begin{cases} 1, & \text{if the } i\text{-th thermal unit is operating during } t, \\ 0, & \text{in other case,} \end{cases} \tag{8}$$

$$z_{jt} = \begin{cases} 1, & \text{if the } j\text{-th thermal unit is operating during } t, \\ 0, & \text{in other case,} \end{cases} \tag{9}$$

$$p_i,\ p_j,\ q_j,\ w_j \ge 0, \ \ \forall\, t \in T,\ i \in \mathcal{S},\ j \in \mathcal{H}, \tag{10}$$

where $\mathbf{E}$ is the mathematical expectation operator, such that $\mathbf{E}(a, b, c) = (\overline{a}, \overline{b}, \overline{c})$, and the operating costs related to each thermal unit includes variable and fixed production costs. The function $a_{it} + b_{it} p_{it} + c_{it} p_{it}^2$, expresses the variable costs, and constant g_i represents the sum of the fixed costs associated to the operation of the i-th thermal unit during interval t. Similarly, constant k_j represents the sum of fixed costs associated to the operation of the j-th hydro plant during period t. In practice, these costs are well identified [20], and can be summarized as: loss of water during maintenance; wear and tear of the windings due to temperature changes during the start-up; wear and tear of mechanical equipment during the start-up; malfunctions in the control equipment during the start-up; and loss of water during the start-up. In this formulation, equation (2) can be viewed as the customer service level.

Thus, for all $t \in T$, and by the properties of the mathematical expectation, equation (1) can be simplified as minimize

$$g(p) = \left[\sum_{i \in \mathcal{S}} (\overline{b}_i p_{it} + \overline{c}_i p_i^2) + \sum_{i \in \mathcal{S}} (g_i - \overline{a}_i) y_i + \sum_{j \in \mathcal{H}} k_j z_j \right]. \tag{11}$$

Assume that the probability density function (pdf) of the random variable D is known for all $t = 1, \dots, T$; and it is given by $f_D(\xi), \forall\, t \in T$. Then, equation (2) is equivalent to

$$\mathbf{P}\left(D_t \leq \sum_{i\in\mathcal{S}} p_{it} + \sum_{j\in\mathcal{H}} p_{jt}\right) = \int_0^{\rho} dF_{D_t}(\xi) = 1 - \alpha, \tag{12}$$

where $\rho = \sum_{i\in\mathcal{S}} p_{it} + \sum_{j\in\mathcal{H}} p_{it}$, and $\alpha \in (0, 1)$.

In particular, if $D_t \sim \mathcal{N}(\mu, \sigma^2)$, with $\mathbf{E}(D_t) = \mu_{D_i}$ and $\mathrm{Var}(D_t) = \sigma^2$, equation (12) can be written as follows

$$\mathbf{P}\left(\mathcal{Z} \leq \frac{\left(\sum_{i\in\mathcal{S}} p_{it} + \sum_{j\in\mathcal{H}} p_{jt}\right) - \mu_{D_t}}{\sqrt{\sigma^2}}\right) = 1 - \alpha, \tag{13}$$

where $\mathcal{Z} \sim \mathcal{N}(0, 1)$. Let K_{α_i} be the standard value such that $F_{D_t}(K_{\alpha_i}) = 1 - \alpha_i$. Note that, expression (13) is satisfied if and only if

$$\frac{\left(\sum_{i\in\mathcal{S}} p_{it} + \sum_{j\in\mathcal{H}} p_{jt}\right) - \mu_{D_t}}{\sqrt{\sigma^2}} \geq K_{\alpha_i},$$

thus, constrain (2) is equivalent to

$$\sum_{i\in\mathcal{S}} p_{it} + \sum_{i\in\mathcal{H}} p_{jt} \geq \mu_{D_t} + \sigma K_{\alpha_i}, \tag{14}$$

The function of water flow through turbines is assumed known and it has the form (See [27])

$$h(p_j) = \beta_0 + \beta_1 p_j + \beta_2 p_j^2, \tag{15}$$

where $\beta_0, \beta_1, \beta_2$ are unknown constants.

Finally, and using the binary variables y and z, equations (5) and (6) can be decomposed as follows

$$p_{it}^{\max} y_{it} - p_{it} \geq 0, \;\; i \in \mathcal{S}, \tag{16}$$

$$p_{it} - p_{it}^{\min} y_{it} \geq 0, \;\; i \in \mathcal{S}, \tag{17}$$

$$p_{jt}^{\max} z_{jt} - p_{jt} \geq 0, \;\; i \in \mathcal{H}, \tag{18}$$

$$p_{jt} - p_{jt}^{\min} z_{jt} \geq 0, \;\; i \in \mathcal{H}. \tag{19}$$

Note that y_{it} and z_{jt}, can be characterized as "one-shot" outlays or fixed charges variables.

4. Numerical Example

To illustrate our proposal we used information from [27] and [11]. We consider 3 hydro plants using Francis turbins and 3 thermal units. The characteristics of the system analyzed are shown in Tables 1 to 4. Table 1 shows the mathematical expectation for each component (a, b, c) the limits of power generation of thermal units and their respective fixed operating costs. Table 2 shows the coefficients proposed for evaluating water requirements as a function of power demand in each turbine and the operating limits of power generation of hydro plants. The corresponding fixed costs are $k_j = 90,000$, $j = 1, 2, 3$. The periods considered and demand parameters are shown in Table 3, where, $Gm = \lfloor \mu_D + \sigma_D K_{\alpha_i} \rfloor$, i.e., the largest integer less than $(\mu_D + \sigma_D K_{\alpha_i})$.

Table 1. Technical characteristics of thermal units.

i	$\overline{a}$	$\overline{b}$	$\overline{c}$	$p_{it}^{\min}$	$p_{it}^{\max}$	g_i
1	561	7.92	0.001562	200	400	79,284
2	310	7.85	0.00194	300	400	105,665
3	780	7.97	0.00482	100	200	20,750

Table 2. Technical characteristics of hydro units.

j	β_0	β_1	β_2	$p_{jt}^{\min}$	$p_{jt}^{\max}$
1	51216.863	1173.829	16.382	10	100
2	50834.983	1082.417	15.551	10	100
3	49816.928	1168.097	12.052	50	150

Table 3. Intervals and demand parameters $(k_{\alpha_i} = 1.96 \text{ for } \alpha = 0.05)$.

t	1	2	3	4	5	6
μ_D	500	520	550	560	610	670
σ_D	20	21	30	20	22	21
Gm	540	562	609	600	654	712
t	7	8	9	10	11	12
μ_D	730	790	820	850	900	1000
σ_D	15	15	18	24	25	20
Gm	760	820	856	898	949	1040
T	13	14	15	16	17	18
μ_D	1100	1150	1200	1210	1215	1225
σ_D	16	18	18	12	10	10
Gm	1132	1186	1236	1234	1235	1245
t	19	20	21	22	23	24
μ_D	1200	1150	1090	980	800	750
σ_D	14	14	15	17	19	20
Gm	1228	1178	1120	1014	838	790

With respect to the water inflows, in literature it is common to use the following random variables to estimate them [17]: (a) Normal distribution, (b) Lognormal distribution (used to describe the flood flows), (c) Gamma distributions (used to model many natural phenomena, including daily, monthly and annual streamflows as well as flood flows) [4], (d) Log-Pearson type 3 distribution (this distribution has found wide use in modelling flood frequencies and has been recommended for that purpose [4, 15]), (e) Gumbel and GEV (Generalized Extreme Value) distributions (in recent years, these have been used as a general model of extreme events including flood flows, particularly in the context of regionalization procedures [14]). For a deeper analysis of the stochasticity from water inflows and energy load see Uhr [21].

In our proposal we use the gamma distribution with pdf, mean and variance given by

$$f_X(x, \alpha, \theta) = x^{\alpha-1} \frac{e^{-x/\theta}}{\theta^{\alpha}\,\Gamma(\alpha)}, \ \mathbf{E}(X) = \alpha\,\theta, \ \mathrm{Var}(X) = \alpha\,\theta^2$$

and to project the simulated value we use the product $(F_X^{-1}(\mu)) \times \varrho$, with $\varrho = 3600$. Here, $(F_X^{-1}(u)) \times u \in \mathcal{U}(0, 1)$ represents the inverse transform of the cumulative distribution function of gamma density. Table 4 shows the operating conditions of the hydro system and the parameters used in the gamma function to estimate the inflows to each reservoir for all $t \in T$.

4.1. Solution schemes

To solve the instance proposed we use two approaches. In the first one, we use a random search technique and for each $t \in T$ we obtain $p^* = (p_i^*, p_j^*)$ such that $g(p^*) \leftarrow \min$. The entire collection of p^* values are an optimized realization of the stochastic process $\xi = \{\xi_t := (p_{it}, p_{jt}, q_{jt}, D_t, w_{jt}, s_{jt}, x_t, z_t)\}_{t=1}^{T}$, $i \in \mathcal{S}$, $j \in \mathcal{H}$. This means that even if the initial condition (or starting point) is known, there are many possibilities the process might go to, but some paths may be more probable and other less so. In our second proposal, we use a genetic algorithm and obtain at once all p^*. Then, using the above mentioned method we optimize the problem described, Figure 2. The flow chart of both algorithms is depicted in Figure 3.

Table 4. Technical characteristics of thermal units, $t = 1, \ldots, 23$.

Hydro units characteristics							Gamma parameters		
j	W_j	$q_j^{\min}$	$q_j^{\max}$	w_{j0}	w_{jt}	$w_{j,24}$	α_j	θ_j	ϱ
1	5.2×10^7	60,000	340,000	45,000,000	40,000,000	40,000,000	1.41	47.92	3600
2	2.1×10^7	60,000	320,000	16,000,000	12,000,000	10,200,000	1.62	47.92	3600
3	5.1×10^6	135,000	500,000	21,000,000	14,000,000	14,000,000	1.28	42.81	3600

Table 5. Comparative aspects of the implementation and optimal solutions.

Method		Hydro system			Thermal system			Total	Storage volume			Total cost	
	t	p_1^*	p_2^*	p_3^*	p_1^*	p_2^*	p_3^*	ρ	w_{1t}	w_{2t}	w_{3t}	$g(p)$	p^*
RSM	1	88	97	132	209	303	0	829	4.5156 E7	1.5945 E7	2.0695 E7	4.5984 E5	
GA	1	0	0	148	0	337	125	610	4.5293 E7	1.6337 E7	2.0794 E7	2.2144 E5	√
RSM	2	119	79	141	368	0	0	707	4.5156 E7	1.5945 E7	2.0695 E7	3.5431 E5	
GA	2	99	0	0	264	0	350	630	4.5055 E7	1.6451 E7	2.0861 E7	2.2209 E5	√
RSM	3	112	94	140	340	0	0	686	4.5140 E7	1.6032 E7	2.0056 E7	3.5378 E5	√
GA	3	0	71	78	373	371	196	1089	4.5192 E7	1.6413 E7	2.0766 E7	3.9545 E5	
RSM	4	120	92	135	362	0	0	709	4.4838 E7	1.6246 E7	2.0069 E7	3.5419 E5	
GA	4	0	43	0	0	397	180	620	4.5423 E7	1.6557 E7	2.0950 E7	3.2252 E5	√
RSM	5	96	97	134	240	0	184	751	4.5065 E7	1.6056 E7	2.0103E7	4.5937 E5	
GA	5	0	0	125	349	0	194	668	4.5988 E7	1.7181 E7	2.1068 E7	1.9606 E5	√
RSM	6	76	87	145	213	312	0	833	4.4804 E7	1.6296 E7	2.0182 E7	4.5998 E5	
GA	6	0	0	149	396	400	125	1070	4.5993 E7	1.7190 E7	2.0624 E7	3.0525 E5	√
RSM	7	91	88	139	219	310	0	847	4.4719 E7	1.6289 E7	2.0714 E7	4.6005 E5	
GA	7	0	100	0	340	341	0	781	4.4609 E7	1.6996 E7	2.0696 E7	2.8160 E5	√
RSM	8	103	94	137	201	383	0	918	4.5473 E7	1.6182 E7	2.0509 E7	4.6046 E5	
GA	8	0	0	150	400	391	0	941	4.6246 E7	1.7185 E7	2.0367 E7	2.8260 E5	√
RSM	9	103	90	143	260	337	0	933	4.5513 E7	1.6266 E7	2.0768 E7	4.6092 E5	
GA	9	71	0	0	287	394	137	889	4.6241 E7	1.7436 E7	2.0536 E7	3.0433 E5	√
RSM	10	96	98	145	238	378	0	955	4.5246 E7	1.6306 E7	2.0596 E7	4.6096 E5	
GA	10	0	0	0	350	369	185	904	4.6417 E7	1.7646 E7	2.0674 E7	2.1511 E5	√
RSM	11	118	77	142	251	396	0	984	4.5024 E7	1.6366 E7	2.0567 E7	4.6133 E5	
GA	11	100	94	0	393	387	0	974	4.6216 E7	1.7517 E7	2.0776 E7	3.7250 E5	√
RSM	12	120	75	138	248	385	0	966	4.5059 E7	1.7000 E7	2.0122 E7	4.6118 E5	
GA	12	99	0	147	247	400	196	1089	4.6068 E7	1.7732 E7	2.0478 E7	3.9460 E5	√
RSM	13	115	69	142	321	397	0	1044	4.4820 E7	1.6877 E7	1.9938 E7	4.6252 E5	
GA	13	95	0	149	400	344	172	1160	4.5894 E7	1.7899 E7	2.0137 E7	3.9521 E5	√
RSM	14	112	98	141	331	369	0	1051	4.5426 E7	1.6824 E7	1.9524 E7	4.6244 E5	
GA	14	100	0	100	400	400	200	1200	4.5746 E7	1.8119 E7	2.0015 E7	3.9601 E5	√
RSM	15	119	98	94	222	361	0	894	4.5329 E7	1.6700 E7	1.9487 E7	4.6056 E5	
GA	15	0	96	150	400	400	197	1243	4.5796 E7	1.7888 E7	1.9600 E7	3.9598 E5	√
RSM	16	116	79	150	326	398	0	1069	4.4997 E7	1.7280 E7	1.9221 E7	4.6262 E5	
GA	16	100	99	145	400	387	198	1329	4.5512 E7	1.7643 E7	1.9204 E7	4.8586 E5	√
RSM	17	90	93	140	360	384	195	1262	4.6309 E7	1.9664 E7	2.0450 E7	5.7052 E5	
GA	17	100	100	144	298	398	200	1240	4.5366 E7	1.7550 E7	1.8925 E7	4.8507 E5	√
RSM	18	108	96	149	348	396	152	1249	4.7720 E7	2.0194 E7	2.1461 E7	5.6999 E7	
GA	18	77	100	150	375	398	150	1250	4.5365 E7	1.7514 E7	1.8664 E7	4.8528 E5	√
RSM	19	103	91	146	347	382	183	1252	4.9004 E7	2.1000 E7	2.2799 E7	5.7014 E5	
GA	19	100	100	150	400	400	200	1350	4.5144 E7	1.7338 E7	1.8298 E7	4.8601 E5	√
RSM	20	117	70	149	388	373	186	1283	5.0193 E7	2.1000 E7	2.4001 E7	5.7088 E5	
GA	20	99	97	125	325	385	175	1206	4.5133 E7	1.7398 E7	1.8202 E7	4.8494 E5	√
RSM	21	115	82	107	396	400	198	1298	5.1632 E7	2.1000 E7	2.5181 E7	5.7141 E5	
GA	21	0	0	150	387	400	198	1135	4.5309 E7	1.7609 E7	1.7889 E7	3.0587 E5	√
RSM	22	118	97	142	392	345	185	1279	5.2000 E7	2.1000 E7	2.7580 E7	5.7069 E5	
GA	22	0	0	148	398	392	172	1110	4.5588 E7	1.7931 E7	1.7671 E7	3.0564 E5	√
RSM	23	107	86	136	228	392	189	1138	5.1953 E7	2.1000 E7	2.7487 E7	5.6828 E5	
GA	23	97	0	0	316	342	193	948	4.5419 E7	1.8113 E7	1.7788 E7	3.0464 E5	√
RSM	24	119	100	116	375	369	0	1079	5.1954 E7	2.0925 E7	2.7280 E7	4.6327 E5	
GA	24	0	99	124	0	394	184	801	4.5689 E7	1.8116 E7	1.7657 E7	3.1253 E5	√

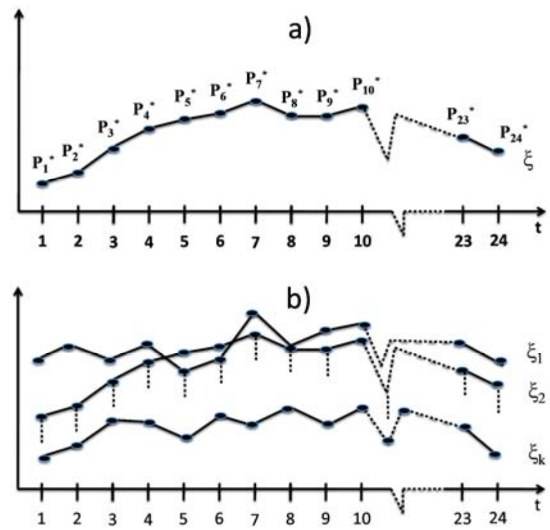


Figure 2. Approaches used: (a) RMS (A single optimum realization), (b) GA (selection of the optimal realization from a total sample).

The formal description of both procedures is as follows.

The random search method (RSM)

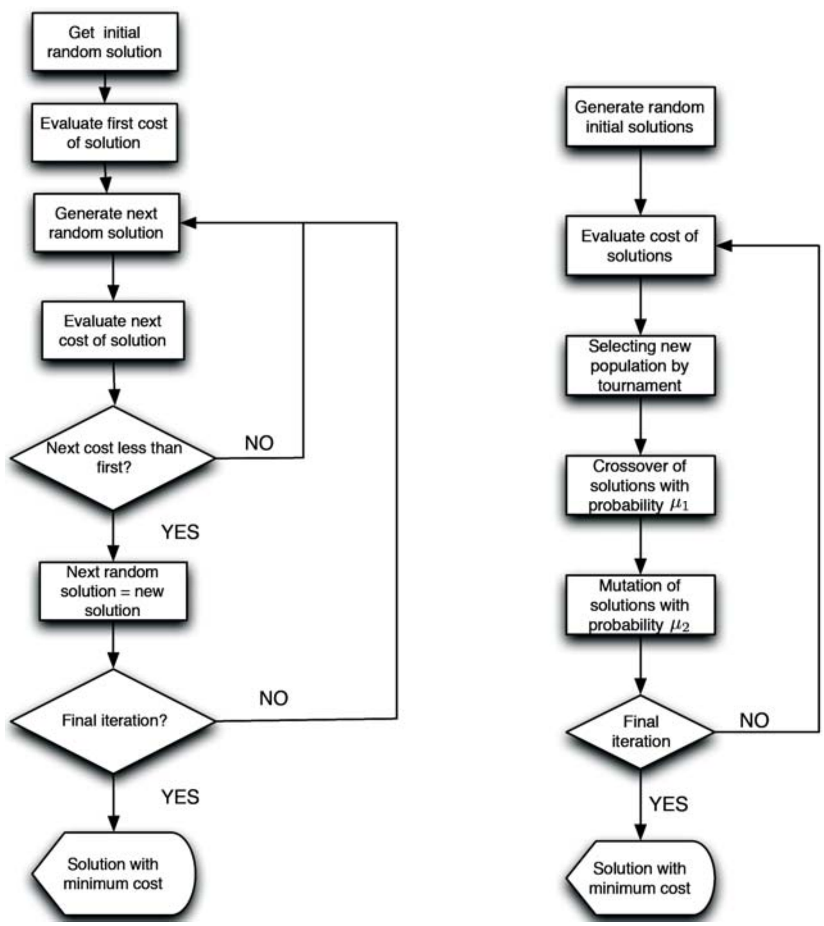
Monte Carlo optimization is a class of algorithms that seek a maximum by sampling, using a pseudo-random number generator. It is a technique for estimating the solution, x of a numerical mathematical problem by means of an artificial sampling experiment. The estimate is usually given as the average value, in a sample, of some statistic whose mathematical expectation is equal to x.

Let Ω be the feasible set of the problem, i.e., any set of all possible solutions $p = (p_i, p_j)$ that satisfy restrictions imposed to the problem in any $t \in T$. Let (η_k) be a sequence of i.i.d. random vectors obtained at random from with uniform probability. In our first approach, the algorithm keeps the best point found so far until a better point is detected:

$$p_{n+1} = \begin{cases} \eta_{n+1}, & \text{if } g(\eta_{n+1}) < g(p_n), \\ p_n, & \text{if } g(\eta_{n+1}) \geq g(p_n), \end{cases} \tag{20}$$

where $p_1 := \eta_1$, and for $n \to \infty$, $p_n \to p^*$ a.s. In this sense, p^* is such that $g(p^*) \leftarrow \min$.

The convergence properties of (20), are widely discussed in Pflug [24]. This algorithm was designed to optimize unconstrained problems, however, we eliminate this inconvenience obtaining samples directly from Ω. Also, restricting the search to integer values, we satisfy the integrity constraint imposed on the model. Although seemingly simple to use, it requires a large number of samples to obtain a feasible optimal solution and time consuming during the process as the search is seeking to the boundary of Ω, i.e., when the solution must meet the higher demand values and close to the generation limits, equations (5) and (6). The average time required by the algorithm using MATLAB is around 354 seconds, to evaluate 100,000 iterations.



(a) (b)

Figure 3. (a) RMS and (b) Genetic algorithm Lay-out.

The genetic algorithm method (GA)

In order to minimize the cost of generating the required demand, a binary genetic algorithm was also implemented with probability μ_1 for crossover, and μ_2, μ_3 for mutation and point mutation probabilities, respectively. Firstly, a set of feasible random initial solutions is generated, each solution represents the decisions about using or not a thermal or an hydro unit, and the amount of power generated by each unit. In this way, every solution represents a sequence of decisions for 24 hours given by the process ξ. To obtain feasible solutions, an initial solution for the i-th hour is randomly generated, this one is checked for reviewing that the minimum required power for this hour is produced, and all restrictions about water and power bounds are fulfilled. If a non-feasible solution is obtained, hence it is discarded, another random one is calculated, and this process is repeated until a feasible solution is produced for the i-th hour. Then, the capacities of reservoirs are updated for the next hour, and the same procedure is achieved to yield other random solution for the $(i+1)$-th hour.

In each hour, every power unit is described by 11 bits; where the first 10 bits represent the power required for this unit and the last one is the binary flag indicating if the unit is online in this hour. In this sense, the complete solution in every hour has associated 66 bits; and every solution is composed by 1584 bits.

In our genetic approach, 40 random solutions are generated, and each are evaluated in the model to obtain its corresponding cost. Tournament is used for selecting the new set of solutions, looking for minimizing the cost of them. With this new set of refined solutions, *a* crossover is performed with a probability of μ_1, between two random solutions, where a single-point crossover is used.

The new chains of bits so obtained are reviewed, in order to know if they are feasible; on the contrary, they are discarded a other 2 new solutions are calculated in the same way until to obtain feasible solutions. These new ones replace the 2 parents previously selected. In this particular problem, since the required demand in every hour is often far from the maximum power generation of the system, there is a high probability of producing feasible solutions in a crossover.

Mutation is similarly achieved, a solution is selected with a probability of μ_2, hence a bit of this solution is inverted with a probability of μ_3; where μ_3; $1/1584$ following the criteria specified in [13]. Once a solution is mutated, its feasibility is checked; in case that a non-feasible solution is obtained; mutation is reversed. This algorithm was also implemented in MATLAB, and it takes about 135 sec. for calculating 40 iterations with 40 solutions.

4.2. Results obtained

The implementation of algorithms (RSM) and (GA) produced the results shown in Table 5. Here, we show comparative aspects of implementation and optimal solution obtained by each algorithm and the optimal solution is highlighted by using the symbol √.

Total costs for each alternative were $10,200,000.00 for RSM and $8,256,568.00 for the GM method. Obviously, the genetic algorithm approach improves significantly the results obtained by the RSM. Naturally, these quantities are only a lottery of the random variable that represents the global cost of the MPS. If the process is repeated κ times, each solution is different, with mean and variance finite. By the above, it is also possible to obtain the probability density function (pdf) of the total cost and its parameter values.

The time required for convergence were also significant. RMS needed 100,000 iterations and the total cost of this alternative is $ 8,256,568.00 which improves the one obtained by Monte Carlo sampling method.

5. Conclusions

In this document we proposed a non-linear stochastic and integer programming model to obtain the MPS of the hydrothermal coordination problem. We use two approaches to solve it. The first algorithm is the well-known procedure to deal with the minimization of functions in this setting, the so called Stochastic Approximation Method that can be viewed as a recursive Monte-Carlo optimization method. In this variant we selected random samples of P from Ω so that, these contain only integer components to secure the integrity of the model.

The other alternative used was a genetic algorithm. For the instances showed, this method was more efficient than RMS either the time required and the accuracy obtained. However, experience showed that the time required to obtain solutions in both algorithms where power demand is approaching the upper limits of generation capacity (equations (2), (5) and (6)) grows significantly.

The approach used in this research, proved to be sufficient but not efficient. However, the application of opyimization is straightforward opening the way for the application of meta heuristics such as simulated annealing, ant colony or particle swarm optimization. Our main contribution in this proposal is the use of reliability functions to ensure that, the power generated meets the average demand with certain probability.

The next activity in this research involves the application of alternative techniques and compare their results (accuracy and speed of convergence) with the ones obtained here.

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