Complex Dynamics in a Hexagonal Cellular Automaton

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ABSTRACT

Hexagonal cellular automata (CA) were studied with interest as a variation of the famous Game of Life CA, mainly for spiral phenomena simulations; where the most interesting constructions are related to the Belousov-Zhabotinsky reaction. In this paper, we study a special kind of hexagonal CA known as the Spiral rule. Such automaton displays a non-trivial complex behaviour related to discrete models of reaction-diffusion chemical media, dominated by spiral guns that easily emerge from random initial conditions. Computing abilities of Spiral rule automata are shown by means of logic gates, defined by collisions between mobile self-localizations. Also, a more extended classification of complex self-localization patterns is presented, including some self-organized patterns.

KEYWORDS: hexagonal cellular automata, gliders, guns, collisions, logic gates, complexity.

1. ANTECEDENTS

Spiral rule is a synchronous totalistic three-state two-dimensional hexagonal CA introduced by Adamatzky and Wuensche in 2005 [13]. Such automaton displays an non-trivial complex dynamics behaviour dominated by mobile and stationary self-localizations (gliders or particles), including the emergence of *spiral guns* producing mobile self-localizations.

In [6] some computing capacities and the fundamental complex activity of the Spiral rule are introduced. Also, a summary of complex structures, basic collisions, and basic properties of the Spiral rule are displayed on Wuensche's home page (are shown in DDLab [7]).[1]

The rule therefore is yet on study. Our interest and contribution in this paper is the implementation of universal logic gates produced by collisions among mobile self-localizations yielded from spiral guns. Besides, a more complete classification of their complex structures is given, consequently reporting new complex patterns in Spiral rule.

2. THE SPIRAL RULE CA

The Spiral rule is a two-dimensional three-state CA evolving on a hexagonal lattice. The hexagonal local function f is a variation of Moore's function as an isotropic $\mathcal{V}$ =7-neighbourhood (Fig. 1), as follows:

$$f(\mathcal{V})^t \rightarrow x_{i,j}^{t+1} \quad (1)$$

Let $\Sigma = \{0, 1, 2\}$ be the alphabet and the local function $f : \Sigma^{\mathcal{V}} \rightarrow \Sigma$ takes into account 2187 neighbourhoods.

[1] http://www.cogs.susx.ac.uk/users/andywu/multi_value/spiral_rule.html

978-1-61284-383-4/11/$26.00

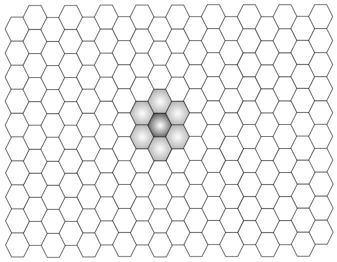

Figure 1. Hexagonal Lattice and a 7-Neighbourhood Respectively.

However, the Spiral rule is a totalistic CA which compresses the number of neighbourhoods by the sum of their states [12]. Thus the Spiral rule is coded by a number of cells in Σ_i on $\mathcal{V}$ as:

$$\begin{array}{l l} \text{766555444433333222222111111100000000} & \Sigma_2 \\ \text{010210321043210543210654321076543210} & \Sigma_1 \\ \text{001012012301234012345012345601234567} & \Sigma_0 \\ \hline \text{000200120021220221200222122022221210} & \end{array}$$

This means, for example, that given seven cells in state 2 and none in state 1 or 0 on $\mathcal{V}$, hence this neighbourhood evolves into 0 in the next generation. For simplicity, we can represent the totalistic code in hexadecimal notation as 020609a2982a68aa64; although looking for a more transparent representation, the totalistic evolution rule can be represented as a triangular matrix [6].

	$\lvert\Sigma_1\rvert$ 0	1	2	3	4	5	6	7
$\lvert\Sigma_2\rvert$ 0	0	1	2	1	2	2	2	2
1	0	2	2	1	2	2	2	
2	0	0	2	1	2	2		
3	0	2	2	1	2			
4	0	0	2	1				
5	0	0	2					
6	0	0						
7	0							

(2)

This matrix representation [6] describes the number of cells in state 1 as columns, the number of cells in state 2 as rows and, the number of cells in state 0 is deduced by $7-(|\Sigma_1|+|\Sigma_2|)$. For example, whether we have three cells in state 2 and two cells in state 1 hence there are two cells in state 0 and the neighbourhood evolves to state 2.

3. COMPLEX DYNAMICS IN SPIRAL RULE

The Spiral rule opens a new universe of complex patterns emerging on the hexagonal evolution space. In this section we present a number of new structures on the Spiral rule CA. Eventually such complex patterns become very useful to develop a number of computing devices, or for another potential engineering devices indeed.

3.1. Mobile Self-Localizations: *Gliders*

The Spiral rule has a great diversity of gliders travelling in the evolution space. These mobile self-localizations (or gliders in CA literature), can be described by a number of particular properties as: mass, volume, period, translation, and speed.

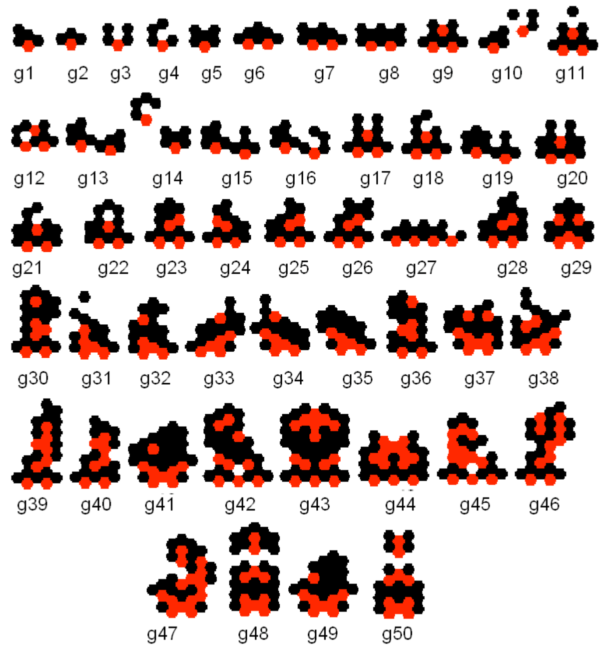

Figure 2. Gliders in Spiral Rule; State 2 is Represented in Black, State 1 in Grey, and the Stable Background (state 0) in white.

We have enumerated 50 gliders, several of them are new with regard to other analysis. Figure 2 displays all the known gliders in the Spiral rule starting from basic or primitive gliders up to large and composed ones (including those which can be extended). Experimentally we have observed that several of them do not have a high probability to emerge from random initial conditions and inclusively survive for few generations, because they are very sensitive to small perturbations.

Table 2 depicts general properties for every glider in the Spiral rule; where *mass* represents the number of cells in state 1 and 2 inside of each volume glider, whether it has more than one form during its period hence the mass is the biggest number of cells. *Period* is the number of evolutions needed for each glider to return to the same shape, *translation* is the number of cells where such pattern move given its period, and finally *speed* of particles is calculated as the rate of its translation between its period.

With such a diversity of gliders, we can refine a clas-

Table 1. Glider Properties in Spiral Rule. State Substrate (State 2), Activator (State 1), and Inhibitor (State 0).

glider	mass	period	translation	speed
g_1	5	1	1	1
g_2	5	2	2	1
g_3	5	2	2	1
g_4	5	2	2	1
g_5	6	1	1	1
g_6	8	1	1	1
g_7	9	1	1	1
g_8	10	1	1	1
g_9	10	1	1	1
g_{10}	10	4	4	1
g_{11}	11	1	1	1
g_{12}	11	4	4	1
g_{13}	11	4	4	1
g_{14}	11	4	4	1
g_{15}	11	4	4	1
g_{16}	11	4	4	1
g_{17}	12	1	1	1
g_{18}	12	2	2	1
g_{19}	12	4	4	1
g_{20}	14	2	2	1
g_{21}	14	2	2	1
g_{22}	14	2	2	1
g_{23}	15	2	2	1
g_{24}	16	2	2	1
g_{25}	16	2	2	1

sification given a family or specie of them. This way, Tab. 3 presents three main branches of species of gliders in the Spiral rule. In particular, *extendible* gliders can be configured and connected as mobile *polymers* over their six possible directions. Extendible glider means that they have extensions inside structure preserving the basic form.

(a) (b)

Figure 3. Still-life Configurations in the Spiral Rule. Also (b) Has All Possible Variations (memory) on its Outer Shell (For Details See [4]).

3.2. Static Stationary Localizations: *Still-Life Configurations*

Spiral rule has basically a pair of basic or primitives static stationary localizations known as still-life configurations in CA. Such patterns can live on the evolution space without alteration; of course, in the lack of any perturbation. Figure 3 displays these still-life patterns that can be also connected as polymers as well to yield extensions of such patterns.

Table 2. Glider Properties in Spiral Rule. State Substrate (State 2), Activator (State 1), and Inhibitor (State 0).

glider	mass	period	translation	speed
g_{26}	16	2	2	1
g_{27}	17	2	2	1
g_{28}	17	4	4	1
g_{29}	17	4	4	1
g_{30}	18	4	4	1
g_{31}	18	4	4	1
g_{32}	19	4	4	1
g_{33}	19	8	8	1
g_{34}	20	8	8	1
g_{35}	22	4	4	1
g_{36}	23	4	4	1
g_{37}	24	4	4	1
g_{38}	25	4	4	1
g_{39}	25	8	8	1
g_{40}	26	8	8	1
g_{41}	29	4	4	1
g_{42}	29	8	8	1

Table 3. Species of Gliders in the Spiral Rule.

specie	glider
primitive	$g_1, g_2, g_3, g_4, g_5, g_{29}$
compound	$g_6, g_7, g_9, g_{10}, g_{12}, g_{13}, g_{14}, g_{15}, g_{16}, g_{17}, g_{19}, g_{27}, g_{35}$
extendible	$g_8, g_{11}, g_{18}, g_{20}, g_{21}, g_{22}, g_{23}, g_{24}, g_{25}, g_{26}, g_{28}, g_{30}, g_{31}, g_{32}, g_{33}, g_{34}, g_{36}, g_{37}, g_{38}, g_{39}, g_{40}, g_{41}, g_{42}, g_{43}, g_{44}, g_{45}, g_{46}, g_{47}, g_{48}, g_{49}, g_{50}$

Firstly, the still life 'e1' (Fig. 3a) has a mass of 12 active cells while the second still life 'e2' (Fig. 3b) has a mass of 13 active cells. The last still life can be used as a counter of binary strings for a memory device [4, 15], producing a family of still-life configurations.

A remarkable characteristic (similar to Life) is that both still-life configurations work as "eaters." An eater is a configuration which generally deletes gliders coming from a given direction. This structure eventually becomes very useful to control a number of signals or values in a specific process. For example, deleting values in a computation, where some bits are not needed anymore.

3.3. Periodic Stationary Localizations: *Oscillators*

Oscillator patterns are able to emerge in the Spiral rule as well; we can see here an interesting diversity of static patterns oscillating periodically. They are frequently a composition of still-life configurations turning ON and OFF bits periodically.

Figure 4 presents six kinds of oscillators in the Spiral rule. They are composed by fundamental still-life configurations connected, all of them oscillating and changing few values in their structures. Thus, it is not complicated to develop more extended and complex oscillators in the Spiral rule.

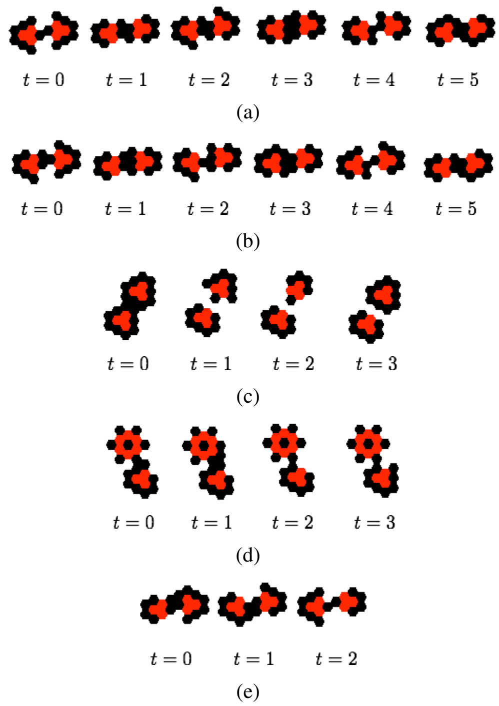

Figure 4. Oscillators Configurations in the Spiral Rule.

Table 4. Oscillator Properties in the Spiral Rule: o1 (a), o2 (b), o3 (c), o4 (d), and o5 (e).

oscillator	mass	period
o_1	20	6
o_2	20	6
o_3	24	4
o_4	24	4
o_5	20	3

Table 4 shows general properties for each oscillator in Fig. 4. Additionally these oscillators are capable to work as eater configurations as well. However, no simple blinkers or flip-flop configurations are still reported.

3.4. Glider Guns

One of the most notable features in the Spiral rule CA is the diversity of glider guns that can appear into its evolution space. A *glider gun* is a complex configuration generating gliders periodically. In the CA literature, the existence of a glider gun also represents the solution of the unlimited-growth problem [8].

The Spiral rule has two types of glider guns: fixed and in movement. A fixed gun cannot change of place and position, while a moving gun can travel along some direction emitting gliders also.

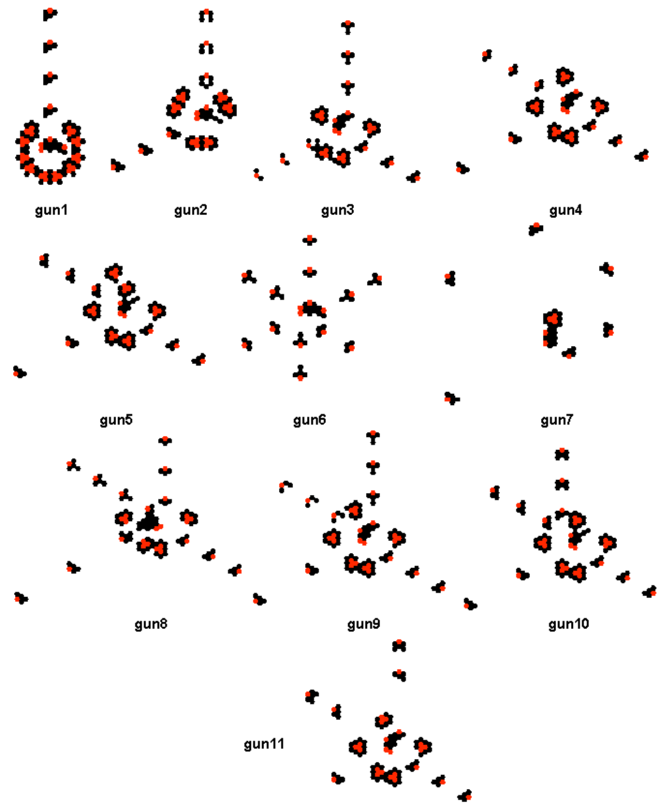

Figure 5. Stationary Glider Guns in the Spiral Rule. A Number of non-Natural Guns are Presented Too.

Table 5 and Fig. 5 show general properties and dynamics of fixed guns evolving in the Spiral rule respectively. The most frequent glider guns produced by the Spiral rule from random initial conditions are gun6 and gun7. They have a high and slow frequency emitting six g_2 and g_1 gliders respectively. While gun6 produces six g_2 gliders every six generations, gun7 yields six g_1 gliders every 22 generations (see Tab. 5). Other gun variations are obtained adding still life or oscillators, affecting the production of gliders or changing their identity and number. Of course, they are not basic guns but they can be modified to yield a different number or kind of gliders and its frequency as well, see guns gun1–gun5, gun8–gun11 to look modified guns.

Particularly stationary glider guns *gun6* and *gun7* (basic guns in Spiral rule) describe characteristic “spiral guns” in chemical reactions, as we can see in Belousov-Zhabotinsky phenomena [3, 2, 9], gliders are CA analogs of wave-fragments (localized excitations) propagating in sub-excitable reaction.

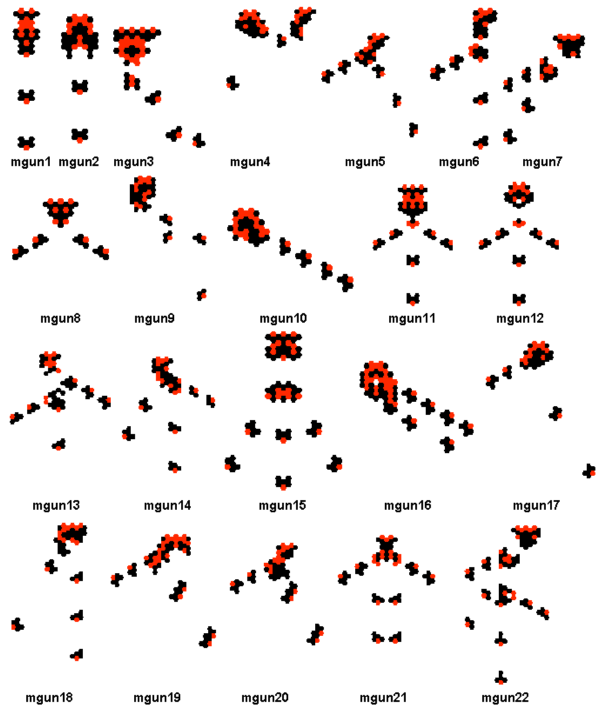

Figure 6. Mobile Glider Guns in the Spiral Rule (First Set).

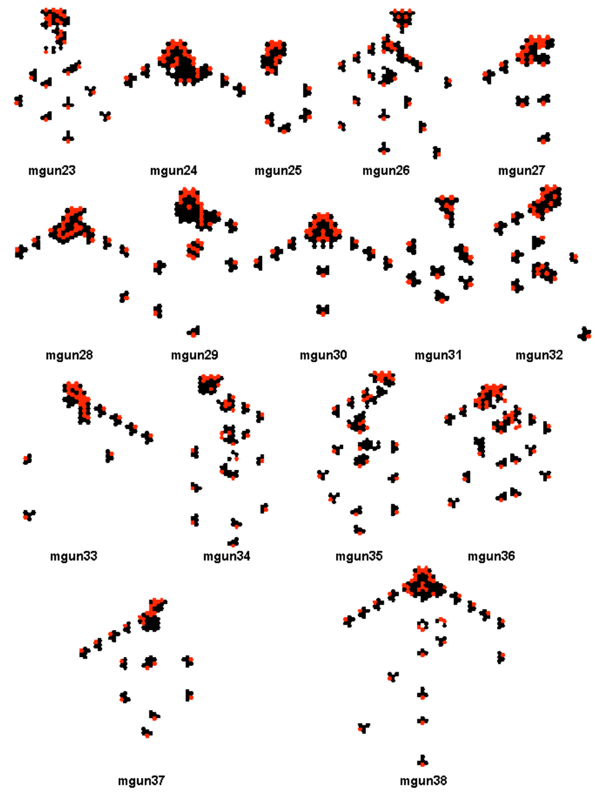

Figure 7. Mobile Glider Guns in the Spiral Rule (Second Set).

Also, the Spiral rule has a large number of mobile glider guns, generally they are formed by complex structures generating more than two gliders. However, the guns are very sensitive to any perturbation, consequently destroying the gun configuration. While natural spiral guns (gun6 and gun7) are very robust to defend their structures from many collisions, there are few of them that are able to destroy these structures as well. Figure 6 and 7 present the broad diversity of mobile glider guns in the Spiral rule, having up to 38 different types.

Thus, there is always a way to yield basic gliders in the Spiral rule from some glider gun.

4. LOGIC GATES AND BEYOND

This section describes constructions to simulate computing devices in the Spiral rule by glider collisions, implementing universal logic gates and other useful computing devices. They are inspired by previous developments as in the Game of Life CA [8]. Thus, the presence of gliders represents bits in state 1 and its complement (absence) represents bits in state 0.

The first construction implementing a logic gate in the Spiral rule was to design a NOT gate.[2] This one presents a NOT gate processing the string $\neg(1111110)$. Here a high frequency spiral gun gun6 produces six gliders where five localizations are suppressed by eaters to preserve only one. Thus the first spiral gun (east position) yields periodically the sequence $1111\ldots$. Hence other two slow frequency spiral guns of kind gun7 generate additional eaters to delete a bit of such sequence, given the string $(1111110)^*$. Finally a fourth spiral gun gun6 (north position) produces the NOT operation obtaining the string $(0000001)^*$ by annihilation reactions.

Gaps between spiral guns can be manipulated to get a desired string. For example, Fig. 8 displays another NOT gate processing the string $\neg(1100110)$.

This way, we have constructed specific initial conditions to simulate: OR (Fig. 9) and AND (Fig. 10) logic gates. Of course, additional still life patterns and spiral guns configurations are needed in each design to synchronize

[2]Please see an animation from http://www.youtube.com/watch?v=_bC5ucq_sKc. The construction was done using DDLab software, to acquire the file 'notGt_sr.eed' please download it from http://uncomp.uwe.ac.uk/genaro/Papers/Thesis.html.

Table 5. Properties of Stationary Glider Guns in the Spiral Rule.

gun	production	frequency	period	volume	gliders emitted
gun1	g_1	1	6	15×15	1
gun2	g_1, g_3	2	6	15×15	2
gun3	g_1, g_2, g_3	3	6	14×15	3
gun4	$3g_1, 2g_2$	5	12	16×17	3
gun5	$5g_1$	5	12	19×17	3
gun6	$6g_2$	6	6	8×9	6
gun7	$6g_1$	6	22	12×12	6
gun8	$3g_1, 4g_2$	7	12	14×14	4
gun9	$3g_1, 2g_2, 2g_4$	7	12	15×17	4
gun10	$5g_1, 2g_5$	7	12	15×15	4
gun11	$13g_1, 4g_5$	17	30	15×17	4

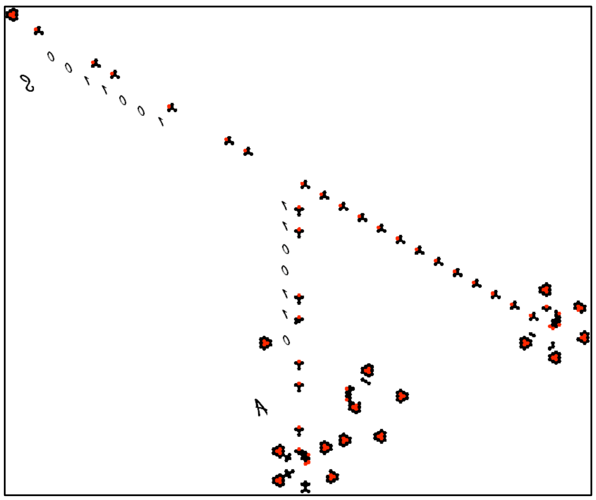

Figure 8. NOT **Gate Implementation in the Spiral Rule String** $\neg(1100110)$**.**

multiple collisions, and controlling a respective sequence of bits.

Additionally, a low frequency DELAY device has been designed as well using a gun1 spiral gun. The goal is to reflect the original input by three reflection reactions which preserve the same sequence at the end. Of course, the delay time can be manipulated increasing the gap among each reflection. Such device is useful to synchronize multiple signals in order to generate more sophisticated computations in further works.

5. FINAL REMARKS

Universal logic gates and other computing devices in the Spiral rule have been implemented, showing the potential of this hexagonal CA to exhibit complex patterns and synchronize multiple collisions.[3] The next step will be

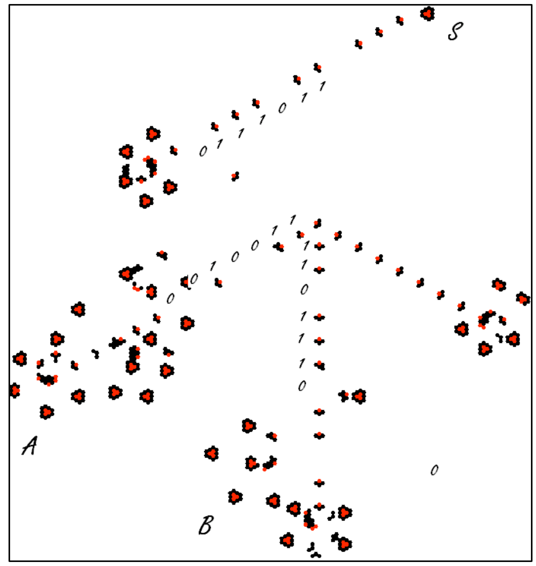

Figure 9. OR **Gate Implemented in the Spiral Rule.**

the design of full logic circuits working together to get a complete implementation for a given computable function. Consequently, it is necessary to engineer of the Spiral rule constructions, based on such logic gates to get a full universality, employing similar constructions done in other hexagonal CA models [11, 5].

About unconventional computing, these results make a bluepring to implement reaction-diffusion computers on Belousov-Zhabotinsky systems [3]. Here, mobile self-localizations are represented as a fragment of waves and their interactions are a scheme for three states: substrate (state 2), activator (state 1), and inhibitor (state 0); where the spiral guns represent a discrete analogy for a classical spiral wave in an excitable media. An interesting study determining spiral forms in CA can be consulted in [10]. The spiral guns can also be related to crystallization computers [1] where a crystallized way will be precisely a glider travelling on such direction. Experimental laboratory tests are working in this direction at the ICUC.[4]

All simulations were done with SpiralSimulator[5], and DDLab[6] software [14].

[3] Implementations of capacitor and reflection devices in Spiral rule can be watched in http://www.youtube.com/watch?v=Cx5QYxvfF9g, and http://www.youtube.com/watch?v=H2xvG-UHM9o.

[4] International Centre of Unconventional Computing, University of the West of England, Bristol, United Kingdom. Home page http://uncomp.uwe.ac.uk/.

[5] Here you can download SpiralSimulator software and source files to reproduce every logic gate designed in this paper http://uncomp.uwe.ac.uk/genaro/Papers/Thesis.html.

[6] Here you can download DDLab software http://www.ddlab.org/.

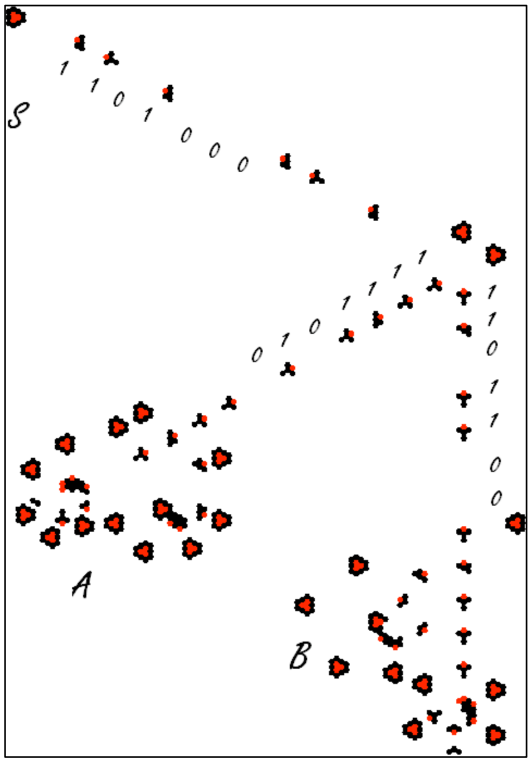

Figure 10. AND **Gate Implemented in the Spiral Rule.**

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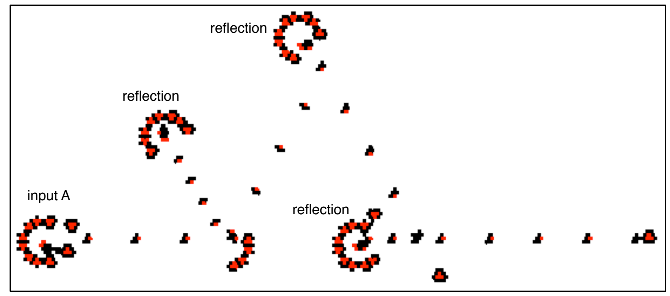

Figure 11. DELAY **Device Implemented in the Spiral Rule.**

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