WAYS OF REASONING AND TYPES OF PROOFS THAT MATHEMATICS TEACHERS SHOW IN TECHNOLOGY-ENHANCED INSTRUCTION

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In this report we document and analyze the ways of reasoning and types of proofs employed by high school mathematics teachers to validate conjectures and to justify procedures in order to solve problems, which emerge when they work in a dynamic geometry environment.

Proof is a fundamental activity in mathematical practice (Hanna, 2000; Weber, 2001), also is a key element in school mathematics (National Council of Teachers of Mathematics, 2000). However, research in mathematics education evidences that students have serious difficulties in understanding and presenting deductive proofs (Harel and Sowder, 1998; Schoenfeld, 1985).

Why do students experience difficulties in constructing mathematical proofs? One reason may be related to the way used to introduce aspects of proofs in mathematical instruction. Balacheff, 2000 argues that instructional methods rely on asking students to imitate their teachers behaviors.

In this context, we consider that in order for students to identify the proving activity as a central in their mathematical experiences, teachers need to have solid understanding of what the concept of proof entails (Stylianides, 2005), not only deductive proof, but also the use of arguments and justifications in general. In this context, it is necessary to carry out research studies that provide information to design instructional strategies and activities that encourage students to use distinct types of mathematical proofs, as well as observing dimensions and aspects that characterize them, this implies to focus on learners through studying teachers' behaviors.

Objectives

The goal of this study was identify the proof schemes (Harel y Sowder, 1998), showed by high school mathematics teachers, when they pose and solve problems using a dynamic geometry software (Cabri Géométre). This is, we were interested in documenting the rationality in which justification processes are based on, and to characterize the ways that teachers used certain type of reasoning when they employed dynamic software as an important part of the activity of doing mathematics.

Theoretical Perspective

The theoretical perspective of this work is based on the construccion of proof schemes (Harel & Sowder, 1998), and theory of problem solving (Polya, 1945; Schoenfeld, 1985). We use Harel and Sowder's taxonomy to explain types of convincing process used by students to construct their proofs (Harel & Sowder, 1998, p. 241) and because the appearance of those schemes is in accordance with the cognitive continuity among the discovery of mathematical relations, conjectures formulation and proof construction.

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Method

Participants in this study were seven high school mathematics teachers (two males and five females), graduate students of mathematics education in México. None of the teachers had used Cabri before participating in this study but they had studied geometry at the high school or at university level.

Starting with either open geometric situation (e.g. simple dynamic configurations) or open problem, teachers were asked to pose problems using Cabri, solved them and justify their observations, as well as solutions procedures. These activities took place during fifteen sessions, ninety minutes each.

Data Sources

The data sources consisted in the Cabri electronic files of the activities developed by each teacher, in weekly written reports, a final report in which the participants were asked to put in writing their conjectures or theorems, and videotaped interviews.

Results

Main results of this work were that teachers often used in consistent way several proof schemes, mainly perceptual proof schemes and inductive proof schemes; likewise, the software supported significantly the use of transformational proof schemes and constructive proof schemes, which are related with the use of heuristics such as "considering a partial solution", "working backwards" and "taking the problem as solved".

Intuitive axiomatic proof schemes were also used in a consistent way, though with significant differences among teachers. Besides, it was identified a low performance in the written formulation of conjectures as conditional sentences; as well as a tendency to associating intuitive-axiomatic proof schemes with conviction on the truth of mathematical facts, leaving aside, apparently, the importance of empirical proof schemes.

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