Commun Nonlinear Sci Numer Simulat 19 (2014) 941–966

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Emergence of density dynamics by surface interpolation in elementary cellular automata

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ARTICLE INFO

Article history:
Received 27 February 2013
Received in revised form 29 May 2013
Accepted 14 August 2013
Available online 24 August 2013

Keywords:
Cellular automata classification
Mean field theory
Basins of attraction
Density behaviour
Surface interpolation

ABSTRACT

A classic problem in elementary cellular automata (ECAs) is the specification of numerical tools to represent and study their dynamical behaviour. Mean field theory and basins of attraction have been commonly used; however, although the first case gives the long term estimation of density, frequently it does not show an adequate approximation for the step-by-step temporal behaviour; mainly for non-trivial behaviour. In the second case, basins of attraction display a complete representation of the evolution of an ECA, but they are limited up to configurations of 32 cells; and for the same ECA, one can obtain tens of basins to analyse. This paper is devoted to represent the dynamics of density in ECAs for hundreds of cells using only two surfaces calculated by the nearest-neighbour interpolation. A diversity of surfaces emerges in this analysis. Consequently, we propose a surface and histogram based classification for periodic, chaotic and complex ECA.

1. Introduction

Elementary cellular automata (ECAs) have been widely studied because they are able to produce interesting dynamical behaviour based on very simple local interactions [1,2].

A classic problem in ECAs has been the specification of numerical tools and methods to analyse, predict and classify their dynamical behaviour.

The use of mean field theory for this task was initiated by Wolfram in [3]. Then, it was developed by Gutowitz [4], McIntosh [5], and Martinez [6] among many others. The idea is to model the long-term behaviour of density with a polynomial specified by the evolution rule of an ECA.

These polynomials are commonly nonlinear and are directly related to the dynamics of ECAs.

Another way to represent the dynamics of ECAs is using basins of attraction [7]. In this case, we have a graphic representation of all evolution in an ECA, and the branches and basins in these graphs describe the dynamics of the system. However, this technique can produce several basins for the same automaton and it is limited up to 32 cells because of the exponential growth.

On the other hand, recent papers have exposed numerical techniques for detecting chaotic behaviour in ECAs. These studies take a sample of evolutions to estimate Lyapunov exponents [8, 9, 10], response curves [11] and Fourier spectra [12]. In particular, ECAs have been analysed in [13] using a Walsh transformation in order to know their efficiency in the generation

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http://dx.doi.org/10.1016/j.cnsns.2013.08.013