Operator Equalities for Singular Integral Operators and Their Applications

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Summary. The article consists of two parts. In the first, we establish a relation between the singular integral operators on the unit circle with a model orientation-reversing shift operator and the singular integral operators on the unit circle with a model orientation-preserving shift operator. In the second part, a Riemann boundary value problem with shift is considered. We reduce this problem to a matrix characteristic singular integral operator without shift.

1 Introduction

We denote the Cauchy singular integral operator along a contour $\mathcal{L}$ by

$$(S_{\mathcal{L}}\varphi)(t) = \frac{1}{\pi i}\int\limits_{\mathcal{L}} \frac{\varphi(\tau)}{\tau - t} d\tau$$

and the identity operator by $(I_{\mathcal{L}}\varphi)(t) = \varphi(t)$

In the paper [1] we constructed a similarity transformation

$$F^{-1}AF = D, \tag{1}$$

between the singular integral operators A with the rotation operator $W_{\mathbb{T}}$ through the angle $2\pi/m$ on the unit circle $\mathbb{T}$, acting on the space $L_2(\mathbb{T})$, and a certain matrix characteristic singular integral operator without shifts acting on the space $L_2^m(\mathbb{T})$.

For $m = 2$, $(W_{\mathbf{T}}\varphi)(t) = \varphi(-t)$,

$$A = a_0 I_{\mathbf{T}} + b_0 S_{\mathbf{T}} + a_1 W_{\mathbf{T}} + b_1 S_{\mathbf{T}} W_{\mathbf{T}}, \quad A \in [L_2(\mathbf{T})],$$

$$D = uI_{\mathbf{T}} + vS_{\mathbf{T}}, \quad D \in [L_2^2(\mathbf{T})]$$

the coefficients $a_0(t), b_0(t), a_1(t), b_1(t)$ are bounded measurable functions.

We denote by $[H_1, H_2]$ the set of all bounded linear operators mapping the Banach space H_1 into the Banach space H_2, $[H_1] \equiv [H_1, H_1]$.