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THE STOCHASTIC SHORT-TERM HYDROTHERMAL SCHEDULING PROBLEM

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Abstract: In this document we develop a non linear stochastic integer model formulation for the unit commitment problem of thermal and hydro units to management demand and optimal short-term operation of a hydrothermal electric facility. We consider a power generation system comprising thermal and hydro units and the problem concerns the scheduling of operation levels for all power units and considering the hydro constrains, such that the operation costs over the time horizon are minimal. The concept of reliability functions is introduced to ensure the meet demand with certain probability. Inflows to reservoirs, cost coefficients and spillage are considered random. We use the Monte Carlo sampling to optimize the instances required each period. We report the practical and theoretical results.

1. INTRODUCTION

The systematic coordination of the operation of a system formed by hydroelectric generation plants is a classical problem involving the planning of the operation of a hydraulic generation system and a thermal system. The generation scheduling problem consists of determining the optimal operation strategy for the next scheduling period, subject to a variety of constrains, in literature this is known as the hydrothermal generation scheduling problem (HGSP) (Gil et al. 2003). The most versions involves the allocation of generation among the hydro-electric and thermal plants so as to minimize the total operation costs of thermal plants while satisfying the various constrains on the hydraulic and power systems network. Usually, the short term period covers from 1 to 7 days, and then, this period is subdivided into smaller time intervals of 1 to 4 hours in which the information of the system is known and the decision variables should be optimized.

This is one of the most important problems associated with the management of a power utility and can be viewed as a problem of production planning, where the good produced is electricity and it is generated from two sources, a hydroelectric generating plant and a thermal power plant. Here, the problem of inventories does not exist because the good produced must be delivered to the customer at the time that it is generated. The master programming scheduling (MPS) is to develop the programming of system operation for each period specifying the state and the generation level of the thermal set, subject to fundamental constrains that must be satisfied such that the covering of each hourly load (demand), satisfaction of spinning reserve requirements and transmission capacity limits, the limited energy storage capability of water reservoirs and other. Under some assumptions (such determinism for example), the mathematical model can be written in terms of a nonlinear objective function subject to a set of linear or nonlinear constrains. In stochastic approach, the model includes some parameters as random variables, which the most representative is the load required. To model the problem more realistically, the load demand the water inflow rate and the reservoir levels of the hydroelectric plants are considered random and therefore the mathematical complexity of the model significantly increases. Anyway, an efficient generation schedule not only reduces the production cost but also increases the system reliability securing valuable reserves, regulating margins, and maximizing the energy capability of the reservoirs, Zoumas et al., (2004).

2. LITERATURE REVIEW

The solution methods of the HGSP problem have been approached from several perspectives, however, literature comprises them in five major areas: a) Lagrangian relaxation, b) Metaheuristic decomposition, c) Bender's decomposition, d) Dynamic programming, e) Mixed integer programming.

The Lagrangian relaxation technique uses the Lagrange multipliers to relax system wide demand and reserve requirements decomposed the main problem into unit-wise subproblems that are much easier to solve. Then, the multipliers are updated at the high level typically using a subgradient method Lu et al., (1998). There are many variants of the technique Zuang and Galiana (1988), Virmani et al., (1989), Yan Guan and Rogan (1993) and (1994), Merlin and Sandrin

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(1983), Aoki et al., (1987), Osman and Laporte (1996), Brannlund et al., (1986); but all they are underpinned by the idea of forming an objective function penalized with model constrains forming the Lagrangian function.

Metaheuristics are a class of approximate methods that have been developed strongly since their inception in the early 1980's. They are designed to optimize complex optimization problems where classical heuristics and optimization methods have failed to be effective and efficient. Metaheuristics include, but are not limited to: constraint logic programming, genetic algorithms, greedy random adaptive search procedures, neural networks, non-monotonic search strategies, problem and heuristic space-search, simulated annealing, tabu search, threshold algorithms and others (Aoki et al., (1987)). In connection with the HGSP, there is an important class of techniques called the heuristic decomposition methods. These, decompose the HGSP problem into hydro and thermal subproblems. The hydro optimization subproblems use either the thermal cost functions or the thermal system marginal cost to efficiently allocate the water resources within the scheduling horizon Zoumas et al., (2004), Osman and Laporte (1996), and Brannlund et al., (1986). Then, the hydro generation and reserve contributions subtracted from the load and reserve requirements, the thermal subproblems solves a standard unit commitment problem.

Benders decomposition is used to solve the multiperiod HGSP problem and is a natural way to decompose it because the 0/1 variable decisions are decoupled from continuous variable decision (Duncan et al., (1985)). In general, the method fixes the start-up and shut-down schedules of the thermal units, while the Benders subproblem solves a multiperiod optimal power flow. Then, the subproblem sends to the master problem marginal information on the goodness of the proposed start-up and shut-down schedule, which allows the master problem to suggest an improved start-up and shut-down schedule and so on (Alguacil and Conejo (1985), Geofrion (1972)) .

In the general approach of the dynamic programming, the problem is decomposed into a thermal subproblem and a hydro subproblem. The algorithm obtains the non discrete states to substitute the discrete states of water volume levels at each time period and then determines an optimal generation schedule while achieving the minimum fuel cost of power system. The spinning reserve of all units provided can satisfy the requirements of the system for any unexpected change in load or loss of maximum on line generation unit (Lasdon (1970), Yang and Chen (1989), Gorenstin et al., (2002), Dillon et al., (1978)).

This paper proposes the use of random coefficients with minimum variance cost (due to the use of short periods of planning) in the objective function, demand as a random variable normally distributed, and water inflow to the reservoirs and spillage are also random variables. An important consideration also include, is the use of a reliability function associated to the power balance equation (customer service level), and the variable and fixed costs of each production unit. Then, this model can be characterized as a nonlinear, stochastic and integer problem.

3. THE MATHEMATICAL MODEL

In the construction of our proposal we use some of the ideas developed in Gröwe and Römish (2005), i.e., we also consider the scheduling of start-up/shutdown decisions and of operation levels for all power units as a stochastic process. Let the planning horizon be discretized into $t \in T$ uniform subintervals, we define the sets S and H, of thermal and hydro units respectively, and for all $i \in S$ and $j \in H$, the notation used is[1]:

t	Time interval index (hour).
a_{it}, b_{it}, c_{it}	Cost coefficients of thermal units, assumed here as random variables.
p_{it}	Power output of ith thermal unit in megawatts (operation level).
p_{it}^{min}	Minimum power output of ith thermal unit in megawatts.
p_{it}^{max}	Maximum power output of ith thermal unit in megawatts.
g_{it}	Fixed operating costs of ith thermal unit in \$/h.
p_{jt}	Power output of jth hydroplant in megawatts (operation level).
p_{jt}^{min}	Minimum power output of jth hydroplant in megawatts.
p_{jt}^{max}	Maximum power output of jth hydroplant in megawatts.
$q_{jt} = h(p_{jt})$	Water flow rate through the turbine during interval t, in m^3/h.
q_{jt}^{min}	Lower bound for the water discharge, during interval t, in m^3/h.
q_{jt}^{max}	Upper bound for the water discharge, during interval t, in m^3/h.

[1] The standard measurement unit of water flow quantities is m^3/s, however, in this document the water flow quantities are expressed in m^3/h to avoid the use of conversion coefficients in equations.

D_t	Energy demand in megawatts, assumed here a random variable.
k_{jt}	Fixed operating costs of jth hydroplant in \$/h.
W_j	Capacity of the jth reservoir in m^3.
$w_{j,t}$	Storage volume of jth reservoir at end of t in m^3.
$w_{j,t}^{min}$	Lower bound of storage volume of jth reservoir at end of t in m^3.
$s_{j,t}$	Spillage rate over jth reservoir during t in m^3/h., assumed here a random variable.
$r_{j,t}$	Water inflow rate of jth reservoir during t, in m^3/h., assumed here a random variable.
v_{jt}	Volume required of jth reservoir at end of t, in m^3.

Then, for each $t \epsilon T$, the mathematical model is

$$\text{Minimize } E_\theta\left[\sum_{i\epsilon S}(a_{it}y_i + b_{it}p_{it} + c_{it}p_{it}^2 + g_{it}y_i) + \sum_{j\epsilon H}k_{jt}z_{jt}\right], \tag{1}$$

Subject to

$$P\left(D_t \le \sum_{i\in S}p_{it} + \sum_{j\in H}p_{jt}\right) = 1-\alpha, i\in S, j\in H, \alpha\in(0,1) \text{ (Power balance)}, \tag{2}$$

$$w_{jt} = w_{j,t-1} - (q_{jt} - s_{jt} + r_{jt})t, \quad j\in H \text{ (Water balance)}, \tag{3}$$

$$q_{jt} = \beta_0 + \beta_1 p_{jt} + \beta_2 p_{jt}^2, \quad j\in H \text{ (Water use rate characteristics)}, \tag{4}$$

$$p_i^{min} \le p_{it} \le p_i^{max}, \quad i\in S \text{ (Operating limits of } i\text{th thermal unit)}, \tag{5}$$

$$q_{jt}^{min} \le q_{jt} \le q_{jt}^{max}, \quad j\in H \text{ (Water flow rate through the turbine limits)}, \tag{6}$$

$$w_{jt} \ge w_{jt}^{min}, \quad j\in H \text{ (Limit of water stored in reservoir } j \text{ at the end of } t), \tag{7}$$

$$y_{it} = \begin{cases}1, & \text{if the } i\text{th thermal unit is operating during } t, i\epsilon S,\\ 0, & \text{in other case,}\end{cases} \tag{8}$$

$$z_{jt} = \begin{cases}1, & \text{if the } j\text{th hydroplant unit is operating during } t, j\epsilon H\\ 0, & \text{in other case}\end{cases} \tag{9}$$

$$w_i, p_i, p_j, q_j \ge 0, \tag{10}$$

where E is the mathematical expectation operator, $\theta = (a,b,c)$ is a random vector such that $E(\theta) = (\bar{a}, \bar{b}, \bar{c})$ and the operating costs related to a thermal unit include variable and fixed production costs. The function $a_{it} + b_{it}\,p_{it} + c_{it}\,p_{it}^2$, expresses the variable costs, and the constant g_i represents the sum of the fixed costs associated to the operation of the ith thermal unit during t. Similarly, the constant k_j represents the sum of fixed costs associated to the operation of the jth hydroplant during the period t. In practice these costs are well identified (Nilsson and Sjelvgren (2002)), and can be summarized as: loss of water during maintenance; wear and tear of the windings due to temperature changes during the start-up; wear and tear of mechanical equipment during the start-up; malfunctions in the control equipment during the start-up; and loss of water during the start-up. In this formulation, equation (2) can be viewed as the customer service level.
Thus, for any $t\in T$, and by the properties of the mathematical expectation, equation (1) can be simplified as

$$\text{Minimize }\left[\sum_{i\epsilon S}(\bar{b}_i p_{it} + \bar{c}_i p_i^2) + \sum_{i\epsilon S}(g_i - \bar{a}_i)\,y_i + \sum_{j\epsilon H}k_j z_j\right], \quad t = 1,\dots,T, \tag{11}$$

Assume that the probability density function (pdf) of the random variable D is known and it is given by $f_D(\xi), \forall\, t \epsilon T$. Then, equation (2) is equivalent to

$$P\left(D_t \leq \sum_{i\in S} p_{it} + \sum_{j\in H} p_{jt}\right) = \int_0^{\rho} dF_D(\xi) = 1-\alpha, \qquad \alpha \in (0,1) \tag{12}$$

where $\rho = \sum_{i\epsilon S} p_{it} + \sum_{j\epsilon H} p_{jt}$

In particular, if $D \sim N(\mu, \sigma^2)$, with $\mathbf{E}(D) = \mu_D$ and $\mathbf{Var}\,(D) = \sigma^2$, equation (11) can be written as follows.

$$P\left(Z \leq \frac{(\sum_{i\epsilon S} p_{it} + \sum_{j\epsilon H} p_{jt}) - \mu_D}{\sqrt{\sigma^2}}\right) = 1-\alpha, \tag{13}$$

where $Z \sim N(0,1)$. Le$K_{\alpha t}$ be the standard value such that $F_D(K_{\alpha t}) = 1 - \alpha_i$. Note that, expression (11) is satisfied if and only if

$$\frac{(\sum_{i\epsilon S} p_{it} + \sum_{j\epsilon H} p_{jt}) - \mu_D}{\sqrt{\sigma^2}} \geq K_{\alpha t},$$

thus, constrain (2) is equivalent to

$$\sum_{i\in S} p_{it} + \sum_{j\in H} p_{jt} \geq \mu_D + \sigma K_{\alpha t}, \tag{14}$$

The function of water flow through turbines is assumed known and it has the form (See Wood and Wollenberg (1996))

$$h(p_j) = \beta_0 + \beta_1 p_j + \beta_2 p_j^2, \tag{15}$$

where $\beta_0, \beta_1, \beta_2$ are unknown constants.

Finally, and using the binary variables y and z, equation (4) and (5) can be decomposed as follows

$$p_{it}^{max} - p_{it} y_{it} \geq 0, \qquad i\epsilon S, \tag{16}$$

$$y_{it}(p_{it} - p_{it}^{min} \geq 0, \qquad i\epsilon S, \tag{17}$$

$$p_{jt}^{max} - p_{jt} z_{jt} \geq 0, \qquad j\epsilon H, \tag{18}$$

$$z_{jt}(p_{jt} - p_{jt}^{min} \geq 0, \qquad j\epsilon H, \tag{19}$$

4. NUMERICAL EXAMPLE

To illustrate our proposal we used information from Wood and Wollenberg (1996) and Loucks and Bee (2005). We consider 3 hydro plants using Francis turbins and 3 thermal units. The characteristics of the system analyzed are shown in Tables (2) to (4). Table (1) shows the mathematical expectation of θ for each component (a, b, c), the limits of power generation of thermal units and their respective fixed operating costs. Table (2) shows the coefficients proposed for evaluating water requirements as a function of power demand in each turbine, the operating limits of power generation of hydro plants and their fixed operating cost. The periods considered and demand parameters are shown in Table (3).

Table 1: Technical characteristics of thermal units.

Unit i	$\bar{a}$	$\bar{b}$	$\bar{c}$	p_{it}^{min}	p_{it}^{max}	g_i
1	561	7.92	0.001562	200	400	79,284
2	310	7.85	0.00194	300	400	105,665
3	78	7.97	0.00482	100	200	20,750

Table 2: Technical characteristics of hydro units.

Unit j	β_0	β_1	$\overline{\beta_2}$	p_{jt}^{min}	p_{jt}^{max}	k_j
1	51216.863	1173.829	16.382	50	120	90,000
2	50834.983	1082.829	15.551	10	100	90,000
3	49816.928	1168.829	12.052	10	150	90,000

Table 3: Intervals and demand parameters ($K_{\alpha i} = 1.96$ for $\alpha = 0.05$).

t	1	2	3	4	5	6	7	8	9	10	11	12
μ_D	800	670	668	675	720	780	800	850	860	900	900	900
σ_D	10	8	9	12	15	15	20	30	32	28	26	26
$\mu_D + \sigma_D K_{\alpha i}$	820	686	686	699	750	810	840	909	923	955	951	951

	13	14	15	16	17	18	19	20	21	22	23	24
μ_D	990	990	850	1000	1150	1210	1214	1225	1240	1245	1100	1050
σ_D	27	21	20	24	26	15	15	16	18	16	14	10
$\mu_D + \sigma_D K_{\alpha i}$	1041	1031	889	1047	1200	1240	1244	1257	1276	1277	1127	1070

With respect to the water inflows, in literature is common to use the following random variables to estimate them (Bobe and Ashkar (1991), IACWD (1982)) : a) Normal distribution, b) Lognormal distribution (used to describe the flood flows), c) Gamma distributions (used to model many natural phenomena, including daily, monthly and annual stream flows as well as flood flows, (IACWD (1982)), d) Log-Pearson type 3 distribution (this distribution has found wide use in modelling flood frequencies and has been recommended for that purpose (IACWD (1982), Hosking and Wallis (1997)), e) Gumbel and GEV (Generalized Extreme Value) distributions (In recent years, these have been used as a general model of extreme events including flood flows, particularly in the context of regionalization procedures (GOVE Hidroelectric Development (2010)). In our proposal we use the gamma distribution with pdf, mean and variance given by

$$f_X(x, \alpha, \theta) = x^{\alpha-1} \frac{e^{-x/\theta}}{\theta^{\alpha}\Gamma(\alpha)}, \ \mathbf{E}(X) = \alpha\theta, \ \mathbf{Var}(X) = \alpha\theta^2 \quad (21)$$

and to project the simulated value we use the product $(F_X^{-1}(u)) \times \varrho$, with $\varrho = 3600$. Here $(F_X^{-1}(u))$, $u \in U(0,1)$ represents the inverse transform of the cumulative distribution function of gamma density. Table (4) shows the operating conditions of the hydro system and the parameters used in the gamma function to estimate the inflows to each reservoir for all $t \in T$.

Table 4: Technical characteristics of thermal units, $t = 1, \ldots, 23$.

Hydro units characteristics							Gamma parameters		
j	W_j	q_j^{min}	q_j^{max}	$w_{j,0}$	$w_{j,t}$	$w_{j,24}$	α_j	θ_j	ϱ
1	5.2×10^7	150,000	500,000	45,000,000	40,000,000	40,000,000	1.41	47.92	3600
2	2.1×10^7	140,000	500,000	16,000,000	12,000,000	10,200,000	1.62	47.92	3600
3	5.1×10^6	100,000	500,000	21,000,000	14,000,000	14,000,000	1.28	42.81	3600

Monte Carlo optimization is a class of algorithms that seek a maximum by sampling, using a pseudo-random number generator. It is a technique for estimating the solution, x, of a numerical mathematical problem by means of an artificial sampling experiment. The estimate is usually given as the average value, in a sample, of some statistic whose mathematical expectation is equal to x. As a first approximation, we generate $\varphi = 100$ random vectors $(p_i,\ p_j)\ \epsilon\ R^6$ containing feasible solutions and then, ordered them to select the lowest. Feasible solutions were obtained under the scheme "here and now". Table (5) shows the optimal MPS for one sequence of 24 hrs.

Table 5: An optimal solution obtained by Monte Carlo sampling method

Hydro system				Termo system			Total	Storage volumen			Total cost
t	p_1^*	p_2^*	p_3^*	p_1^*	p_2^*	p_3^*	ρ	w_{1t}	w_{2t}	w_{3t}	$g(p)$
1	88	97	132	209	303	0	829	45156097.72	15945837.88	20695336.95	459,843.24
2	119	79	141	368	0	0	707	45156097.72	15945837.88	20695336.95	354,313.88
3	112	94	140	340	0	0	686	45140387.99	16032810.17	20056662.95	353,782.47
4	120	92	135	362	0	0	709	44838183.17	16246749.62	20069122.52	354,197.95
5	96	97	134	240	0	184	751	45065120.47	16056126.87	20103801.78	459,379.18
6	76	87	145	213	312	0	833	44804304.03	16296334.41	20182240.40	459,982.67
7	91	88	139	219	310	0	847	44719135.25	16289213.02	20714623.16	460,052.56
8	103	94	137	201	383	0	918	45473154.95	16182718.32	20509411.93	460,463.11
9	103	90	143	260	337	0	933	45513187.46	16266613.30	20768735.61	460,929.89
10	96	98	145	238	378	0	955	45246629.63	16306052.93	20596405.56	460,963.23
11	118	77	142	251	396	0	984	45024348.36	16366587.77	20567803.24	461,333.82
12	120	75	138	248	385	0	966	45059659.40	17000254.17	20122423.05	461,183.66
13	115	69	142	321	397	0	1044	44820407.61	16877648.01	19938379.10	462,523.03
14	112	98	141	331	369	0	1051	45426772.86	16824458.46	19524150.19	462,442.67
15	119	98	94	222	361	0	894	45329118.55	16700476.76	19487504.11	460,563.73
16	116	79	150	326	398	0	1069	44997328.70	17280566.94	19221139.74	462,622.55
17	90	93	140	360	384	195	1262	46309322.18	19664068.90	20450200.24	570,527.45
18	108	96	149	348	396	152	1249	47720729.67	20194828.68	21461092.08	569,997.43
19	103	91	146	347	382	183	1252	49004902.58	21000000	22799384.98	570,144.75
20	117	70	149	388	373	186	1283	50193503.87	21000000	24001180.09	570,885.59
21	115	82	107	396	400	198	1298	51632331.87	21000000	25181604.15	571,417.21
22	118	97	142	392	345	185	1279	52000000	21000000	27580788.33	570,697.44
23	107	86	136	228	392	189	1138	51953426.05	21000000	27487774.95	568,285.56
24	119	100	116	375	369	0	1079	51954872.34	20925351.19	27280450.19	463,276.36

5. CONCLUSSIONS

In this document we proposed a non linear stochastic and integer programming model to obtain the MPS of the hydrothermal coordination problem. We use a random search technique based on Monte Carlo sampling to optimize the given instance. The problem was programed in Excel and Math Lab to evaluate the instances generated. Experience showed that the time required to obtain solutions where power demand is approaching the upper limits of generation capacity (equations (2), (5) and (6)) grows significantly. In our results the water inflow to dams was greater than the needs of water flow through the turbines; this caused spillages in reservoirs 1 and 2.

The approach used in this research, proved to be sufficient but not efficient. However, opening the way for the application of meta heuristics such a genetic algorithms or ant colony. Our main contribution in this proposal is the use of reliability functions to ensure that, the power generated meets the average demand with certain probability. The use of fixed and variables costs and the consideration of that, the water inflow rate and the corresponding spillage rate are random variables.

The next activity in this research involves the application of alternative techniques and to compare their results (accuracy and speed of convergence) with obtained here.

6. REFERENCES

A. J. Wood and B. F. Wollenberg, (1996), Power Generation Operation and Control, 2nd ed. NY: John Wiley & Sons, Inc.

A.J. Wood and B.F. Wollenberg, (1984), Power Generation Operation and Control, New York, Wiley.

A.M. Geofrion. Generalized Benders, (1972), Decomposition. *Journal of Optimization Theory and Applications*, JOTA, Vol. 10, No. 4, pp. 237 - 260.

B. Bobe. and F. Ashkar., (1991), The gamma distribution and derived distributions applied in hydrology. Littleton Colo., Water Resources Press.

B.G. Gorenstin., N. M. Campodonico., J. P. Costa., and M. V. F. Pereira, (May 2002), Stochastic Optimization of a Hydro-thermal System Including Network Constraints, *IEEE Trans. Power Syst*., vol. 7, pp. 791 - 797.

C. E. Zoumas., A.G. Bakirtzis., J.B. Theocharis., and V. Pertridis, (May. 2004), A Genetic Algorithm Solution Approach to the Hydrothermal Coordination Problem, *IEEE Transactions on Power Systems*, vol. 19, No. 2, pp. 1356 -1364.

D. P. Loucks and E. Van Bee, (2005), Water Resources Systems Planning and Management: An Introduction to Methods, Models and Applications, Studies and Reports in Hydrology, ISBN 92-3-103998-9, UNESCO PUBLISHING.

E. Gil., J. Bustos., and H. Rudnick., (Nov. 2003), Short-Therm Hydrothermal Generation Scheduling Model Using a Genetic Algorithm, *IEEE Transactions on Power Systems*, vol. 18, No. 4, pp. 1256 -1264.

F. Zhuang and F. D. Galiana, (May. 1988), Toward a More Rigorous and Practical Unit Commitment by Lagrangian Relaxation, *IEEE Transactions on Power Systems*, vol. 3, No. 2, pp. 763 - 773.

H. Brannlund., J. A. Budenko., D. Sjelvgren., and N. Andersons, (Nov. 1986), Optimal short term operation planning of a large hydrothermal power system based on a nonlinear network flow concept, *IEEE Trans. Power Syst*., vol. PWRS-1, pp. 75-82.

H. Yan., P. B. Luh., X. Guan., and P. M. Rogan, (Aug. 1996), A flexible approach to short-term hydrothermal coordination. Part II: Dual problem solution procedure, *IEEE Trans. Power Syst*., vol. 11, pp. 1572 - 1578.

H. Yan., P. B. Luh., X. Guan., and P. M. Rogan, (May 1994), Optimization-based scheduling of hydrothermal power systems with pumpedstorage units, *IEEE Trans. Power Syst*., vol. 9, pp. 1023 -1031.

H. Yan., P. B. Luh., X. Guan., and P. M. Rogan, (Feb. 1999), Short-term hydrothermal coordination by Lagrangian relaxation: Solution of the dual problem, *IEE Trans. Power Syst*., vol. 14, pp. 89 – 95.

I.H. Osman., G. Laporte., (1996), Metaheuristics: A bibliography, Annals of Operations Research, 63, pp. 513 - 623.

IACWD (Interagency Advisory Committee on Water Data), (1982), Guidelines for determining flood flow frequency, Bulletin 17B. Reston, Va., US Department of the Interior, US Geological Survey, Office of Water Data Coordination.

J. F. Benders, (1962), Partitioning procedures for solving mixed-variables programming problems, *Numerische Methematik*, vol. 4, pp. 238- 252.

J. Yang and N Chen, Short Therm Hydrothermal Coordination Using Multi-Pass Dynamic Programming, *IEEE Trans. Power Syst.*, vol. 4, pp. 1050 - 1056, Aug. 1989.

J.R.M. Hosking and J.R. Wallis, (1997), Regional frequency analysis: an approach based on L-moments. Cambridge, Cambridge University Press.

K. T. Aoki., K. Satoh., and M. Itoh, (Nov. 1987), Unit commitment in a large-scale power system including fuel constrained thermal and pumped storage hydro, *IEEE Trans. Power Syst*., vol. PWRS-2, pp. 1077 - 1084.

L.S. Lasdon, (1970), Optimization Theory of Large Systems, *Optimization for Large Systems*. New York, Mc. Millan.

Merlin and P. Sandrin, (May 1983), A new method for unit commitment at electricite de france, IEEE Trans. *Power App. Syst.*, vol. PAS-102, pp. 1218 -1225.

N. Alguacil., and A.J. Conejo, (Feb. 2000), Multiperiod Optimal Power Flow Using Benders Decomposition, *IEEE Transactions on Power Systems*, vol. 15, No. 1, pp. 1256 -1264.

N. Gröwe-Kuska, and W Römish., (2005), Stochastic Unit Commitment Power Production Planning, in “Applications of Stochastic Programming”, edited by Stein W. Wallace and William T. Ziemba, MPS-SIAM Series on Optimization, pp. 633 - 653, Philadelphia, PA. EEUU.

Nilsson, O. and Sjelvgren, D., (Aug. 2002), Hydro unit start-up costs and their impact on the short term scheduling strategies of Swedish power producers, *IEEE Trans. Power Syst*., vol. 12, Issue 1, pp. 38 - 44.

P. B. Luh., D. Zhang., R.N Tomastik, (May 1998), An Algorithm for solving the Dual Problem of Hydrothermal Scheduling, *IEEE Transactions on Power Systems*, vol. 13, No. 2, pp. 593 -600.

R. A. Duncan., G. E. Seymore., D. L. Streiffert., and D. J. Engberg, (May 1985), Optimal hydrothermal coordination for multiple reservoir river systems, *IEEE Trans. Power App. Syst.*, vol. PAS-104, pp. 1154-1161.

S. Ruzic., N. Rajakovic., and A. Vuckovic, (Aug. 1996), A flexible approach to short-term hydrothermal coordination. Part I: Problem Formulation and general solution procedure, *IEEE Trans. Power Syst*., vol. 11, pp. 1564 -1571.

S. Virmani., E. C. Adrian., K. Imhof., and S. Mukherjee, (Aug. 1993), Implementation of a Lagrangian relaxation based unit commitment problem, IEEE Trans. Power Syst., vol. 4, pp. 1373-1380, Nov. 1989. H. Yan, P. B. Luh, X. Guan, and P. M. Rogan, Scheduling of hydrothermal power systems, *IEEE Trans. Power Syst.*, vol. 8, pp. 1358-1365.

T. S. Dillon., K. W. Edwin., H.D. Kochs., and R. J. Tand, (Nov/Dic. 1978), Integer Programming Approach to the Problem of Optimal Unit Commitment whit probabilistic reserve determination, *IEEE Trans. Power App. Syst.* PAS 97, No. 6, pp. 2154 - 2166.