




## Leader-follower scheme for unicycle-type mobile robots with mounted camera Esquema líder-seguidor para robots móviles tipo-uniciclo con cámara montada

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### Resumen

Este artículo describe una propuesta de control de formación para robots móviles tipo-uniciclo bajo el esquema líder-seguidor con cámara montada. El objetivo de control es determinado directamente en el espacio de imagen, por lo que esta propuesta corresponde a la alternativa de Control Servo-Visual Basado en Imagen (IBVS por sus siglas en inglés). Utilizando la teoría de estabilidad de Lyapunov, dos controladores son diseñados: uno que depende parcialmente de los parámetros del sistema de visión y otro que es robusto a los parámetros tridimensionales de las características que son mapeadas al espacio de imagen (las características de imagen). Finalmente, se realizan experimentos satisfactorios sobre una plataforma de tiempo-real blando para validar la teoría propuesta.

**Palabras Clave:** Control de formación, robots móviles tipo-uniciclo, control servo-visual basado en imagen, validación experimental.

### Abstract

This paper describes a formation control proposal for unicycle-type mobile robots under the leader-follower scheme with mounted camera. The control objective is determined directly in image space, hence this proposal corresponds with the Image-Based Visual Servoing (IBVS) alternative. Using the Lyapunov stability theory, two controllers are designed: one that depends partially on the parameters of the vision system and another one that is robust to the three-dimensional parameters of the features that are mapped to the image space (the image features). Finally, experiments on a soft real-time platform with satisfactory results are carried out to validate the proposed theory.

**Keywords:** Formation control, unicycle-type mobile robots, image-based visual servoing, experimental validation.

### 1. Introduction

A group of mobile robots working in coordination can perform some complex tasks more effectively and efficiently than just a single robot. Among these complex tasks we have, for example, applications in surveillance, exploration, rescue, search and transportation of materials. Typically, these tasks are performed on unstructured environments, in such a way that it is necessary to measure the internal states of the robots (using proprioceptive sensors) and the workspace (through exteroceptive sensors). Commonly, the employed exteroceptive sensors are global positioning systems (GPSs), RADAR systems (through electromagnetic radio waves), LIDAR (by laser detection) and vision systems (Benhimane *et al.*, 2005; Dani *et al.*, 2009). It can be noted that, due to recent

technological advances, vision systems are already widely used.

The formation control problem consists of moving a group of robots in a coordinated and collaborative manner to perform a specific task. There are different schemes for the formation control problem: the Behavior-Based scheme (Antonelli *et al.*, 2006) where different behaviors, such to keep the formation or to track a target, are imposed on each robot; the Virtual Structure scheme (Belta and Kumar, 2002) that considers the group of robots as a single virtual rigid; and the Leader-Follower scheme (Consolini *et al.*, 2006; Shao *et al.*, 2005) wherein one or more robots are designated as the leaders of the formation and the rest of them as the followers, which must keep desired postures relative to the leaders. In each scheme, the knowledge of the position and orientation (posture) of the

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robots (relative to each other or respect to a global coordinate system) plays an important role.

This paper describes a proposal for the formation control problem of two unicycle-type mobile robots with mounted camera under the leader-follower scheme. Therefore, this proposal is, in turn, framed within the visual servoing field; which has been conceptualized as the use of a vision system to determine the motion of a robotic system (since the original work of Hill and Park (1979)). In the field of visual servoing there are basically two alternatives: the Position-Based one, where the image sequence are used to reconstruct the three-dimensional workspace; and the Image-Based one, where the control objective is given directly in image space. The latter alternative increases the possibility to do not explicitly depend on the extrinsic (which are related to the camera posture) or intrinsic (those that have to do with the camera internal structure) parameters of the vision system (Hutchinson *et al.*, 1996). The camera-mounted arrangement consists in that one or more cameras are installed on the robots and the fixed-camera arrangement involves one or more cameras placed conveniently in the workspace to observe the group of robots.

Some works related to the formation control of mobile robots in the leader-follower scheme within the visual servoing field will be explained below. In Bugarin and Aguilar-Bustos (2014) is addressed this problem with the fixed-camera arrangement and with the Image-Based alternative where it is highlighted that their controller is independent of the knowledge of the vision system parameters. In Benhimane *et al.* (2005), Min *et al.* (2009), and Soria *et al.* (2006) the formation control problem is solved using the camera-mounted arrangement with the Position-Based alternative and, in their results, there exist the need of the full or partial knowledge of the vision system parameters. In Das *et al.* (2002), Mariottini *et al.* (2007), and Roberti *et al.* (2011) are proposed controllers under the same conditions as above (mounted-camera and Position-Based alternative) but by means of catadioptric vision systems, basically to expand the field of view. In Dani *et al.* (2009) is also described a work with the position-based alternative and the camera-mounted arrangement but eliminating the need of the knowledge of the vision system parameters. In Vidal *et al.* (2003) the authors also worked on a camera-mounted proposal but with the Image-Based alternative using a well calibrated catadioptric vision system (that is, the parameters of the vision system are needed) on each follower robot; it must be noted that they estimate the leader velocity using a static three-dimensional point in the workspace and, for the case of two robots (one follower and one leader), it is ensured the no collision between them, however, the robotic system in closed loop presents degenerative configurations (to coping this situation, the authors also illustrate another controller that only guarantees input-to-state stability of the formation). In Wang *et al.* (2019) is used a mounted stereo camera arrangement and a Image-Based alternative to solve a flexible leader follower formation problem. In Guo *et al.* (2017) is proposed an unified Image-Based control scheme for both omnidirectional and perspective cameras. In Li *et al.* (2019) and Liang *et al.* (2018) the authors work with an Image-Based control scheme estimating the velocity of the leader robot and showing bounded or semiglobal stability.

## 2. Problem formulation

Consider Figure 1 to formulate the formation control problem. Observe that the leader robot has two spheres, the spheres  $a$  and  $b$ , which will give the necessary image features for the movement of the follower robot (who has a mounted camera). The radius of each sphere is arbitrary but convenient so that the vision system on the follower robot can process their centroids. Also, note that several coordinate frames are placed: the world frame  $\Sigma_W$  fixed somewhere in the workspace and at a convenient height; the camera frame  $\Sigma_C$  with origin at the center of its lens (with the optical axis  $C_3$  and the plane  $C_1 - C_2$  parallel and perpendicular to the motion plane of the robots, respectively); the follower robot frame  $\Sigma_{O_s}$  with origin coincident with the origin of the camera frame and vertical axis  $O_{s3}$  intersecting the center of the axle that links its two wheels; and the leader robot frame  $\Sigma_{O_l}$  with vertical axis  $O_{l3}$  crossing the center of the axle that links its two wheels. The planes  $W_1 - W_2$ ,  $C_1 - C_3$ ,  $O_{l1} - O_{l2}$ , and  $O_{s1} - O_{s2}$  are all parallels and have the same height.

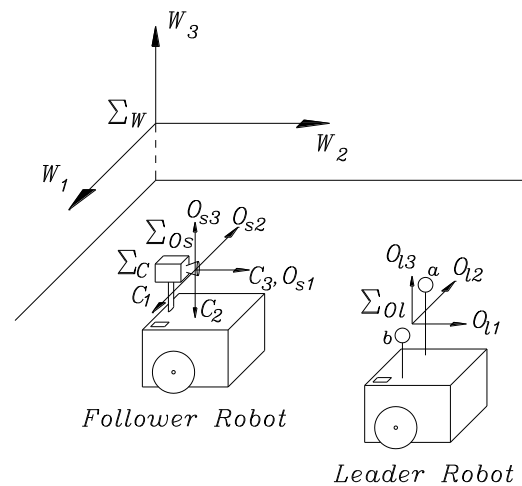


Figure 1: Robotic system scheme.

### 2.1. Vision system model

The mounted camera on the follower robot is modeled as a thin lens camera, this model corresponds to (Kelly and Reyes, 2000)

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \frac{\alpha\lambda}{x_{c3}-\lambda} \begin{bmatrix} x_{c1} \\ x_{c2} \end{bmatrix} + \begin{bmatrix} u_0 \\ v_0 \end{bmatrix}, \quad (1)$$

which represents the mapping of a three-dimensional point  $x_c = [x_{c1} \ x_{c2} \ x_{c3}]^T$  in the scene (respect to the camera frame  $\Sigma_C$ ) to the image plane  $y = [y_1 \ y_2]^T$ ; where  $\alpha$  is the conversion factor from metres to pixels,  $\lambda$  is the lens focal length, and

$$\begin{bmatrix} u_0 \\ v_0 \end{bmatrix} = \alpha O_l + \begin{bmatrix} u'_0 \\ v'_0 \end{bmatrix}$$

with  $u'_0 \in \mathbb{R}$ ,  $v'_0 \in \mathbb{R}$  and  $O_l \in \mathbb{R}^2$  possible camera internal misalignments (constant values). The extrinsic parameters of the camera are conformed by the parameters that define the space location of the camera frame  $\Sigma_C$  and its intrinsic parameters are  $\lambda$ ,  $u'_0$ ,  $v'_0$ ,  $\alpha$  and the components of the vector  $O_l$ .

Therefore, it is considered that the camera produces a two dimensional image of the three-dimensional space through a mapping based on perspective projection and coordinates transformations, so that, evidently, this mapping depends on the characteristic intrinsic and extrinsic camera parameters.

### 2.2. Unicycle-type robot model

The unicycle-type mobile robots are composed by two independently actuated conventional wheels on the same axis and a third wheel without actuator to maintain its horizontal balance. These robots have the following kinematic model (Canudas de Wit *et al.*, 1996; Dixon *et al.*, 2001)

$$\frac{d}{dt} \begin{bmatrix} x_{Wi1} \\ x_{Wi2} \\ \theta_i \end{bmatrix} = \begin{bmatrix} \cos(\theta_i) & 0 \\ \sin(\theta_i) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_{i1} \\ u_{i2} \end{bmatrix}, \quad i \in \{l, s\} \quad (2)$$

where  $l$  and  $s$  are subscripts to denote correspondence with the leader and the follower robot, respectively. The vector  $x_{Wi} = [x_{Wi1} \ x_{Wi2} \ 0]^T$  denotes the position of the unicycle robot  $i$  (the midpoint of the axis joining the two wheels at a conveniently null height) and  $\theta_i$  represents the orientation of the unicycle robot  $i$ , both respect to  $\Sigma_W$ . The components of the input vector  $u_i = [u_{i1} \ u_{i2}]^T$  are the magnitude of the linear (along  $O_{i1}$ ) and angular (around  $O_{i3}$ ) velocity of the robot  $i$ , respectively.

### 2.3. Robotic system model in image space

The robotic system is seen as a nonlinear control system composed by the model of the vision system and the kinematic model of the robots. For this purpose, consider the centroid mapping of the spheres  $a$  and  $b$  located on the leader robot (that is,  $x_{Oa} = [0 \ x_{Oa2} \ x_{Oa3}]$  and  $x_{Ob} = [0 \ x_{Ob2} \ x_{Ob3}]$ , respectively, expressed in the  $\Sigma_{Ol}$  frame) to the image plane ( $y_a \in \mathbb{R}^2$  and  $y_b \in \mathbb{R}^2$ , respectively); that is

$$y_i = \begin{bmatrix} y_{i1} \\ y_{i2} \end{bmatrix} = \frac{\alpha\lambda}{x_{Ci3}-\lambda} \begin{bmatrix} x_{Ci1} \\ x_{Ci2} \end{bmatrix} + \begin{bmatrix} u_0 \\ v_0 \end{bmatrix}, \quad i \in \{a, b\} \quad (3)$$

where  $i \in \{a, b\}$  is a subscript to denote the correspondence with the leader robot spheres.

Note that the necessary coordinate transformation between the world frame and the camera frame can be performed by

$$x_{Ci} = R_W^C(\theta_s)[x_{Wi} - O_W^C], \quad i \in \{a, b\} \quad (4)$$

where  $x_{Wi} = [x_{Wi1} \ x_{Wi2} \ x_{Wi3}]^T$  is the corresponding centroid respect to  $\Sigma_W$ ,  $O_W^C = O_W^{Os} = x_{Ws}$  is the position vector of the origin of  $\Sigma_C$  (or equivalently of  $\Sigma_{Os}$ , see Figure 1) respect to  $\Sigma_W$  and

$$R_W^C(\theta_s) = \begin{bmatrix} \sin(\theta_s) & 0 & \cos(\theta_s) \\ -\cos(\theta_s) & 0 & \sin(\theta_s) \\ 0 & -1 & 0 \end{bmatrix}$$

is the rotation matrix of  $\Sigma_C$  respect to  $\Sigma_W$ . On the other hand

$$x_{Wi} = R_W^{Ol}(\theta_l)[x_{Oi} + O_W^{Ol}], \quad i \in \{a, b\} \quad (5)$$

where  $O_W^{Ol} = x_{Wl}$  is the position vector of the origin of  $\Sigma_{Ol}$  respect to  $\Sigma_W$  and

$$R_W^{Ol}(\theta_l) = \begin{bmatrix} \cos(\theta_l) & -\sin(\theta_l) & 0 \\ \sin(\theta_l) & \cos(\theta_l) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

is the rotation matrix of  $\Sigma_{Ol}$  respect to  $\Sigma_W$ .

Now, with the intention to simplify the analysis, consider that

$$x_{C3} = x_{Ca3} = x_{Cb3} \gg \lambda, \quad (6)$$

namely, that the depths or distances along the optical axis from the camera to the centroid of each sphere are equal and that these distances are much greater than the lens focal length (reasonable hypotheses for relatively large distances between robots).

In this way, if it is defined  $y_{ab} = [y_{ab1} \ y_{ab2}]^T = y_a - y_b$ , from (6) and by means of (3), we can get the following expressions:

$$y_a = \frac{\alpha\lambda}{x_{C3}} \begin{bmatrix} x_{Ca1} \\ x_{Ca2} \end{bmatrix} + \begin{bmatrix} u_0 \\ v_0 \end{bmatrix} \quad (7)$$

and

$$y_{ab} = \frac{\alpha\lambda}{x_{C3}} \begin{bmatrix} x_{Ca1} - x_{Cb1} \\ x_{Ca2} - x_{Cb2} \end{bmatrix}.$$

Finally, after some algebraic manipulations, using (2), (4) and (5), the time derivative of (7) results in the following system (considered the open loop system for control purposes)

$$\dot{y}_a = A(\mu_y; \mu_p)u_s + B(\mu_y; \mu_p)u_l \quad (8)$$

where

$$A(\mu_y; \mu_p) = \begin{bmatrix} -\frac{y_{ab2}[y_{a1}-u_0]}{\alpha\lambda x_{Oab3}} & \alpha\lambda + \frac{[y_{a1}-u_0]^2}{\alpha\lambda} \\ -\frac{y_{ab2}^2 x_{Oa3}}{\alpha\lambda x_{Oab3}^2} & \frac{y_{ab2} x_{Oa3} [y_{a1}-u_0]}{\alpha\lambda x_{Oab3}} \end{bmatrix},$$

$$B(\mu_y; \mu_p) = \begin{bmatrix} \frac{y_{ab1}[y_{a1}-u_0]}{\alpha\lambda x_{Oab2}} & -\frac{y_{ab1} x_{Oa2} [y_{a1}-u_0]}{\alpha\lambda x_{Oab2}} \\ \frac{y_{ab1} y_{ab2} x_{Oa3}}{\alpha\lambda x_{Oab2} x_{Oab3}} & -\frac{y_{ab1} y_{ab2} x_{Oa2} x_{Oa3}}{\alpha\lambda x_{Oab2} x_{Oab3}} \end{bmatrix},$$

with  $\mu_y$  the set of the measured variables in the image plane ( $y_a$  and  $y_{ab}$ ) and  $\mu_p$  the set of the vision system parameters ( $\alpha$ ,  $\lambda$  and  $u_0$ ) and the three-dimensional parameters respect to  $\Sigma_{Ol}$  of the spheres centroid ( $x_{Oa}$  and  $x_{Ob}$ , with  $x_{Oab} = x_{Oa} - x_{Ob}$ ).

It is worth noticed that  $\det\{A(\mu_y; \mu_p)\} = \frac{x_{Oa3}}{x_{Oab3}^2} y_{ab2}^2$ , so if one sphere is placed higher than the other one and  $x_{Oa3} \neq 0$  then there exist the inverse of  $A(\mu_y; \mu_p)$ .

### 3. Image-based visual servo controllers

This section describes the proposed Image-Based visual servo controllers. To this aim, consider the following control objective

$$\lim_{t \rightarrow \infty} e(t) = 0 \quad (9)$$

where  $e = y_{ad} - y_a$  is the formation error and  $y_{ad}$  is the desired position (constant) of the centroid of the sphere  $a$  in image space.

Hence, the first proposed controller that solves the problem of formation control just described is

$$u_s = A(\mu_y; \mu_p)^{-1} [K_p e - B(\mu_y; \mu_p) u_l] \quad (10)$$

where  $K_p$  is the gain matrix that must be symmetric and positive definite.

For the stability proof, substitute the control law (10) into the system (8), such that it is obtained the following equation for the closed loop system

$$\dot{e} = -K_p e,$$

which corresponds to a linear and time invariant (or autonomous) system and because by design  $K_p = K_p^T > 0$  the origin is its only equilibrium state with the property to be globally exponentially stable (see for example Khalil (2001)). In this way, it is formally demonstrated that the formation control objective (9) is satisfied.

It can be noted that neither  $A(\mu_y; \mu_p)$  nor  $B(\mu_y; \mu_p)$  depends explicitly on the parameter  $v_0$  of the vision system, so it is considered that the controller (10) depends partially on the vision system parameters (it depends partially on the intrinsic parameters, however, it does not depend on the extrinsic ones). Also, it must be observed that, in this control proposal, it is necessary the transmission of the leader robot velocities to the follower robot (the works Vidal *et al.* (2003) and Liang *et al.* (2018) are aimed in the direction of estimating these velocities to eliminate the need of their transmission; in the present paper, this is considered as a future work) and that the controller depends on the three-dimensional parameters of the centroids of the spheres in the frame  $\Sigma_{Ol}$ ; fortunately these parameters are directly determined by the user.

In the above analysis, it has been considered the robotic system as a continuous and deterministic one, so the velocity of the leader robot  $u_l$  required in (10) must be continuously differentiable. Furthermore, and for practical reasons, because this result demonstrates exponential stability, if the velocities of the leader robot are relatively small and the initial configuration allows that the spheres be within the field of view of the camera on the follower robot then it is ensured, to some extent, that the spheres do not leave this field of view and that there will not be problems of collision between the robots (a formally proof of these situations are also considered here as a future work).

#### 3.1. Robust Image-Based visual servo controller

It is proposed other Image-Based visual servo controller in this subsection to solve (9) that includes only estimations of

certain three-dimensional parameters related to the centroids of the spheres on the leader robot, which is expressed by

$$u_s = \hat{A}(\mu_y; \mu_p)^{-1} [K_p e + K_i \xi - \hat{B}(\mu_y; \mu_p) u_l] \quad (11)$$

$$\dot{\xi} = e$$

where  $K_p = K_p^T > 0$ ,  $K_i = K_i^T > 0$ ,  $\xi \in \mathbb{R}^2$  is a state vector added to the system,

$$\hat{A}(\mu_y; \mu_p) = \begin{bmatrix} -\frac{y_{ab2}[y_{a1}-u_0]}{\alpha \lambda x_{Oab3}} & \alpha \lambda + \frac{[y_{a1}-u_0]^2}{\alpha} \\ -\frac{y_{ab2}^2 \hat{x}_{Oa3}}{\alpha \lambda x_{Oab3}^2} & \frac{y_{ab2} \hat{x}_{Oa3} [y_{a1}-u_0]}{\alpha \lambda x_{Oab3}} \end{bmatrix},$$

$$\hat{B}(\mu_y; \mu_p) = \begin{bmatrix} \frac{y_{ab1}[y_{a1}-u_0]}{\alpha \lambda \hat{x}_{Oab2}} & -\frac{y_{ab1} \hat{x}_{Oa2} [y_{a1}-u_0]}{\alpha \lambda \hat{x}_{Oab2}} \\ \frac{y_{ab1} y_{ab2} \hat{x}_{Oa3}}{\alpha \lambda \hat{x}_{Oab2} x_{Oab3}} & -\frac{y_{ab1} y_{ab2} \hat{x}_{Oa2} \hat{x}_{Oa3}}{\alpha \lambda \hat{x}_{Oab2} x_{Oab3}} \end{bmatrix},$$

$\hat{x}_{Oa2} = \gamma_1 x_{Oa2}$ ,  $\hat{x}_{Oa3} = \gamma_2 x_{Oa3}$  and  $\hat{x}_{Oab2} = \gamma_3 x_{Oab2}$  (with  $\gamma_1$ ,  $\gamma_2$  and  $\gamma_3$  estimation factors).

This controller (11) substituted into the system (8) results in the following closed loop system

$$\dot{e} = -A(\mu_y; \mu_p) \hat{A}(\mu_y; \mu_p)^{-1} [K_p e + K_i \xi - \hat{B}(\mu_y; \mu_p) u_l] - B(\mu_y; \mu_p) u_l$$

$$\dot{\xi} = e,$$

which can be simplified to

$$\dot{e} = -K_{p\gamma} e - K_{i\gamma} \xi + [I_\gamma \hat{B}(\mu_y; \mu_p) - B(\mu_y; \mu_p)] u_l \quad (12)$$

$$\dot{\xi} = e$$

where  $I_\gamma = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{\gamma_2} \end{bmatrix}$ ,  $K_{p\gamma} = K_p I_\gamma$  and  $K_{i\gamma} = K_i I_\gamma$ .

For the stability analysis of (12), consider its linearization for constant velocities  $u_l = u_l^*$ . Now, do the following change of variables

$$\xi^* = \xi - K_{i\gamma}^{-1} [I_\gamma \hat{B}^*(\mu_y; \mu_p) - B^*(\mu_y; \mu_p)] u_l^*,$$

where  $\hat{B}^*(\mu_y; \mu_p)$  and  $B^*(\mu_y; \mu_p)$  are the respective linearized matrices around the operation point; so that (12) can be expressed by

$$\dot{e} = -K_{p\gamma} e - K_{i\gamma} \xi^* \quad (13)$$

$$\dot{\xi}^* = e.$$

If  $\gamma_2 > 0$ , the only equilibrium state of (13) is the origin  $[e^T \ \xi^{*T}]^T = 0 \in \mathbb{R}^4$ ; note that both  $K_{p\gamma}$  and  $K_{i\gamma}$  remain symmetric and positive definite. Now, consider the following Lyapunov function candidate

$$V(e, \xi^*) = \frac{1}{2} e^T e + \frac{1}{2} \xi^{*T} K_{i\gamma} \xi^*,$$

whose derivative with respect to time along the trajectories of (13) is

$$\dot{V}(e, \xi^*) = e^T [-K_{p\gamma} e - K_{i\gamma} \xi^*] + \xi^{*T} K_{i\gamma} e$$

$$= -e^T K_{p\gamma} e \leq 0.$$

Hence, the origin of (13) is stable and the states are bounded. We can continue with the LaSalle Theorem (see Khalil (2001)) since system (13) is autonomous, in such a way that we have

$$\begin{aligned}\Omega &= \{[e^T \quad \xi^{*T}]^T \in \mathbb{R}^4: \dot{V}(e, \xi^*) = 0\} \\ &= \{e = 0 \in \mathbb{R}^2, \xi^* \in \mathbb{R}^2\}.\end{aligned}$$

Then, in order to a solution  $[e^T(t) \quad \xi^{*T}(t)]^T$  belongs to  $\Omega$  for all  $t \geq 0$ , it is necessary and sufficient that  $e(t) = 0$  for all  $t \geq 0$ , which means that also  $\dot{e}(t) = 0$  for all  $t \geq 0$ . Consequently, from (13), if  $e(t) \in \Omega$  for all  $t \geq 0$  we have

$$0 = -K_{i\gamma} \xi^*$$

and

$$\dot{\xi}^* = 0,$$

therefore, also  $\xi^*(t) = 0$  for all  $t \geq 0$ . That is, the origin of (13) is the only initial condition in  $\Omega$  for which  $[e^T(t) \quad \xi^{*T}(t)]^T \in \Omega$  for all  $t \geq 0$ . This allows us to formally conclude that the origin of (13) is asymptotically stable. Hence the control objective (9) is satisfied at least locally.

#### 4. Experiments

As an important part of this work, the proposed theory has been experimentally validated. For this purpose, two unicycle-type robots model YSR-A, from the Yujin company, have been utilized. These robots receive the commands for their velocities through a RF transmitter that must be connected to a PC via a serial port. In the experiment, it is installed in the same PC a TV card with wireless receiver to get the video signals from a NTSC camera (model QSMCC-MINI color Q-SEE) mounted on the follower robot. Additionally, and just to corroborate the proposed theory, it is installed another video camera that is fixed in the workspace to observe (and measure) the motion of the robots. This additional camera is digital and works at high-speed; its model is UF-1000CL from the UNIQ company and is connected via Camera Link through a Leonardo videoprocessor card of the Arvoo company. In such a way that, the control system and the image processing are implemented on a Pentium IV PC with the RTLinux operating system and under the scheme of soft real-time computer system at a sampling rate of 33 milliseconds. The image processing is performed using binary segmentation and image feature tracking (the centroids of the spheres).

The scenario for the experiment corresponds to the robotic system just described with the robust Image-Based visual servo controller (11) and gains:

$$K_p = \begin{bmatrix} 12 & 0 \\ 0 & 0.6 \end{bmatrix} \text{ and } K_i = \begin{bmatrix} 12 & 0 \\ 0 & 5 \end{bmatrix}.$$

The vision system parameters were:  $\alpha = 72000$  [pixels/m],  $\lambda = 0.0075$  [m],  $u_0 = 160$  [pixels],  $v_0 = 120$  [pixels] (the latter parameter is not required in the implementation of the controller and, in fact, only two values are needed: the product  $\alpha\lambda$  and  $u_0$ ). The spheres were placed in the leader robot with the following (approximate) coordinates of their centroids

$$x_{0\alpha} = [0 \quad 0.0110 \quad -0.0165]^T \text{ [m]}$$

and

$$x_{0b} = [0 \quad -0.0110 \quad 0.0065]^T \text{ [m]}.$$

The vector of the leader robot velocities transmitted to the follower robot corresponds to

$$u_l = \left[ 0.05 \frac{m}{sec} \quad 0.07 \frac{rad}{sec} \right]^T$$

and the formation reference was  $y_{ad} = [114 \quad 167]^T$  [pixels].

The initials conditions were:

$$\begin{aligned}x_{wl}(0) &= [-0.0171 \quad -0.2127 \quad 0]^T \text{ [m]}, \\ x_{ws}(0) &= [-0.1951 \quad -0.1725 \quad 0]^T \text{ [m]}, \\ \theta_l(0) &= -4.2971^\circ, \text{ and} \\ \theta_s(0) &= -8.8506^\circ.\end{aligned}$$

Figure 2(a) shows the trajectories of the mobile robots (dashed line for the leader robot and solid line for the follower robot). Figure 2(b) illustrates the robot orientations versus time (dashed line for the leader robot and solid line for the follower robot). Figure 2(c) describes the evolution respect to time of the formation error norm. And Figure 2(d) presents the graphs of the follower robot velocities (the control laws) versus time (dashed line the linear velocity and solid line the angular velocity). As can be seen, the performance of the experiment is very satisfactory. The follower robot remains in formation with the leader robot with an average of the formation error norm of 8 [pixels].

#### 5. Conclusions

It has been presented a solution proposal for the formation control problem of unicycle-type mobile robots in the leader-follower scheme with mounted camera. The control objective is determined directly in image space and two controllers have been designed: one that needs only two values of the vision system parameters and the centroids of the two spheres required for the calculation of the image features (note that these centroids are determined directly by the user) and another controller that is robust to these centroid parameters. Both Image-Based visual servo controllers are analyzed by the Lyapunov stability theory guaranteeing the achievement of the formation control objective. It is worth noticing that the controllers (in addition to measurements in the image space) require the knowledge of the velocities of the leader robot which could be estimated or solved by some adaptation law (which for purposes of this paper, it is considered as a future work). Finally, we have presented experiments on a soft real-time platform to validate the proposed theory with satisfactory results.

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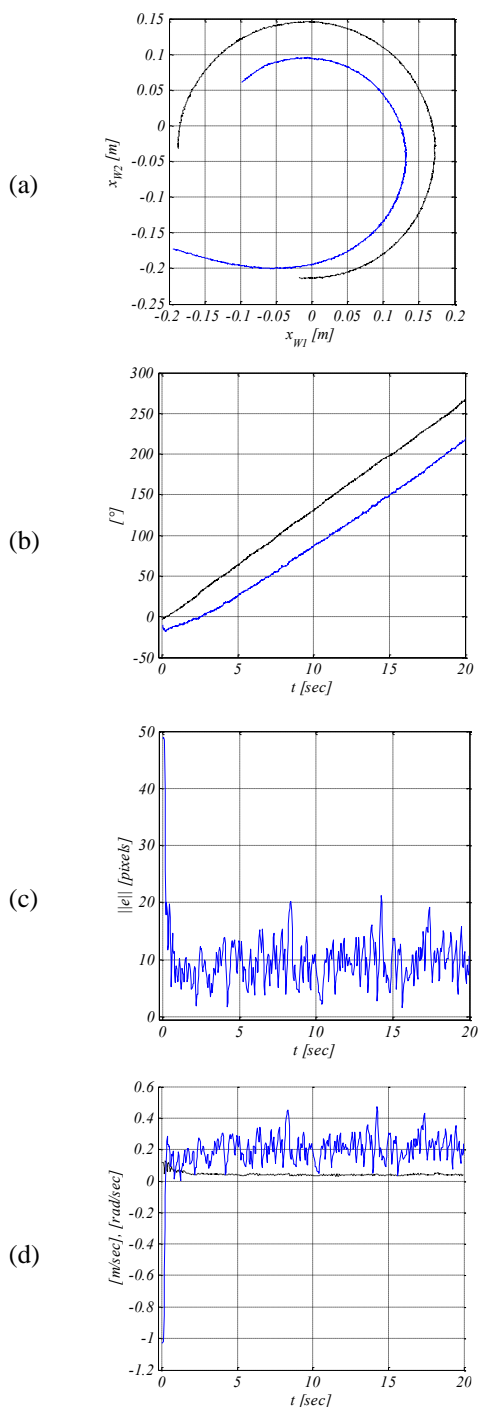


Figure 2: Experimental graphs: (a) trajectories of the mobile robots (dashed leader robot and solid follower robot), (b) robot orientations versus time (dashed leader robot and solid follower robot), (c) formation error norm versus time, and (d) follower robot velocities (control laws) versus time (dashed linear velocity and solid angular velocity).

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