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Modeling of brushed PMDC motor embedded analog velocity servo actuators
Modelado de servo accionamientos analógicos de velocidad con un motor PMDC embebido

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Abstract

This paper deals with the modeling of velocity servo actuators built around brushed permanent magnet direct current motors which are utilized as driven elements in low cost robotics and mechatronics. More specifically, a simplified linear velocity servo model is proposed. It is shown that proportional-integral control is effective for regulating the shaft speed of permanent magnet direct current motors globally under disturbances free situations. This is the main argument for justifying the proportional-integral dominant feedback control approach at the core of most velocity servo actuators.

Keywords: Servo actuators, control, robotics, mechatronics, velocity.

Resumen

En este artículo se aborda el modelado de servo accionamientos de velocidad construidos a partir de motores de corriente directa con imán permanente con escobillas, los cuales se utilizan como elementos de accionamiento en robótica y mecatrónica de bajo costo. De manera más específica, se propone un modelo lineal simplificado de un servo de velocidad. Se demuestra que el control proporciona–integral es efectivo para regular globalmente la velocidad del eje del motor de corriente directa con imán permanente bajo situaciones libres de perturbación. Este es el argumento principal para justificar el control realimentado proporcional–integral dominante en el núcleo de los servo actuadores de velocidad.

Palabras Clave: Servo actuadores, control, robótica, mecatrónica, velocidad.

1. Introduction

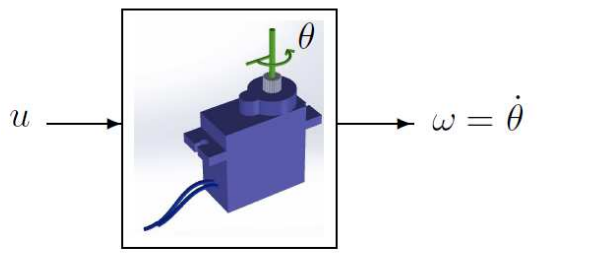

Figure 1: Input-Output sketch of a velocity servo actuator.

Servo actuators are integrated mechatronic drives (see Fig. 1) with an electrical power input u and a feedback controlled mechanical power output represented by the servo actuator shaft displacement θ or its speed $\omega = \dot{\theta}$. The drive combines an embedded electrical motor —commonly a low power PMDC motor—, power and control electronics as well as sensor elements into a compact functional unit.

The two basic types of servo actuators are:

- **Position servos:** widely used for reduced-scale mechanisms and robotics apparatuses (e.g., desktop robot arms and small scale walking robots) where moderate precise motion is required. The shaft of position servo actuators can rotate within limited range. They have physical stops into the gear mechanism to prevent turning beyond these limits to protect the internal rotational servo sensors. They do not provide speed control neither continuous rotation.

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- **Velocity servos** or continuous rotation servos: unlike position servos, the velocity servos shaft can turn continuously at varying speed ω depending upon the commanded input servo signal u.

Velocity servo actuators, also called speed servos for short, are involved implicitly in important theoretical and practical robotics issues such as:

- Kinematic control of robot manipulators (Siciliano, 1990) where the robot inputs are assumed to be the robot joint velocities.
- Orthodox visual servoing (Hashimoto, 1993).
- Kinematic model of wheeled mobile robots where the robot inputs are the wheels speed.

With regard to Figure 2, a low power velocity servo actuator is formed by the following internal hardware:

① Electrical motor, normally a Permanent Magnet Direct Current (PMDC) one.

② A gearbox to reduce the speed of the motor but increasing its available torque.

③ Position or speed sensor for the servo shaft angular position θ or speed ω measurement.

④ Embedded analog or digital controller electronics board (typically a Proportional–Integral (PI) dominant PID controller) which processes the exogenous desired shaft velocity — set-point— u and the current angular speed ω to compute the armature voltage v for the internal PMDC motor ①.

⑤ Servo shaft.

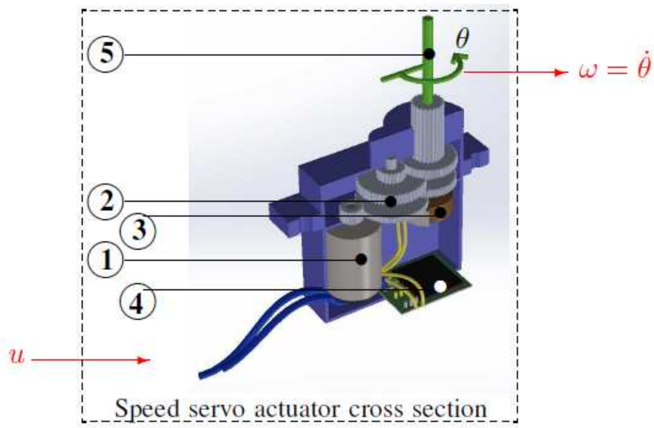

Figure 2: Velocity servo actuator components.

PMDC motors (see Fig. 3) mainly consists of two parts: a stator and an armature. As stated by (Ogata, 2002, p. 96): “Armature–controlled PMDC motors” are controlled —motor control input— by the armature voltage, say $v \in \mathbb{R}$.

PMDC motors do not need field excitation arrangement and they are cheaper and economical for fractional kW applications. So, PMDC motors are extensively used such as in automobiles starter, wipers, washers, hot blowers, air conditioners, toys, and in many more. Small PMDC motors are the core of popular position and velocity servo actuators utilized in low and medium cost robotics.

The goal of this paper is to provide a dynamic model of analog velocity servo actuators equipped with an embedded PMDC motor under PI regulation. The proposed model finds its application in mechatronics and robotics devices. With regard to position servo actuators model, the reader is referred to (Kelly et al., 2021).

Before introducing the proposed velocity servo model, the paper continues by revisiting a model of servo embedded armature–controlled PMDC motors. This is a key ingredient to develop the velocity servo model.

2. A simplified armature–controlled brushed PMDC motor model

For a simplified dynamic model of PMDC motors this paper adapts it from the textbooks (Golnaraghi and Kuo, 2010, Eq. 4-207, p. 202), (Khalil, 2002, p. 425), and (Ogata, 2002, p. 96).

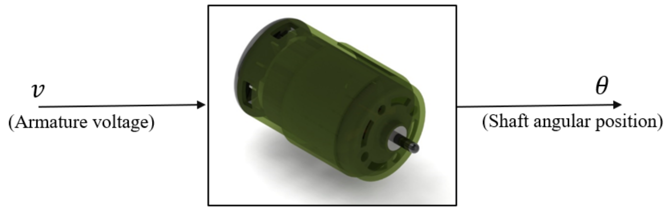

Figure 3: Input-Output sketch of an armature controlled PMDC motor.

In summary, based mainly upon textbook of (Ogata, 2002, Eqs. (3-46) and (3-47), p. 96), an armature-controlled PMDC motor can be modeled by the system (Kelly et al., 2021):

$$J\frac{\mathrm{d}^2}{\mathrm{d}t^2}\theta + b_0\dot{\theta} + \tau_d = ki, \tag{1}$$

$$L\frac{\mathrm{d}}{\mathrm{d}t}i + Ri + k_b\dot{\theta} = v, \tag{2}$$

where the system control input is the armature voltage v applied at the armature terminals and one considers the motor output as its rotor displacement (shaft position) θ (see Fig. 3). Internal variable i stands for the armature current. Exogenous variable τ_d denotes mechanical load torque disturbances input. Remaining (positive and constant) motor parameters are:

$R \neq 0$	armature resistance,
$J > 0$	rotor inertia,
L	armature inductance,
k	torque constant,
k_b	back–emf constant,
b_0	viscous–friction coeficient.

System model (1) and (2) yields the third order differential equation in the dependent variable θ:

$$J\frac{d^3}{dt^3}\theta + b_0\frac{d^2}{dt^2}\theta + \dot{\tau}_d = \frac{k}{L}[v - Ri - K_b\dot{\theta}]. \tag{3}$$

On the other hand in small PMDC motors, typically the armature inductance L is "small", so neglecting L in (2), one solves:

$$0 = v - k_b\dot{\theta} - Ri,$$

to obtain the armature current:

$$i = \frac{v - k_b\dot{\theta}}{R},$$

which in its turn is substituted in (1), giving a second-order linear model structure (Kelly and Moreno, 2001):

$$JR\ddot{\theta} = k\ v - (kk_b + Rb_0)\,\dot{\theta} - R\tau_d. \tag{4}$$

Under disturbance free case ($\tau_d = 0$), this is a commonly used simplified second-order linear structure model of DC motors (Kelly and Moreno, 2001) which can be also rewritten in function of rotor speed $\omega \triangleq \dot{\theta}$ as a first-order linear system structure (Khalil, 2002, p. 425):

$$JR\dot{\omega} = -(kk_b + Rb_0)\,\omega + k\ v - R\tau_d. \tag{5}$$

The associated transfer function (disturbances included) of system (5) is:

$$\omega(s) = G_1(s)V(s) - G_2(s)\tau_d(s), \tag{6}$$

where s is the Laplace complex variable and

$$G_1(s) \triangleq \frac{k}{[JRs + (kk_b + Rb_0)]}, \tag{7}$$

and

$$G_2(s) \triangleq \frac{R}{k}G_1(s).$$

3. Analog Velocity servo actuators

The speed regulation control objective aims to ensure that a body or a rotor shaft angular speed ω tends asymptotically to an arbitrary but constant desired speed ω_d. More formally, from automatic control systems viewpoint, the speed regulation control objective is to achieve:

$$\lim_{t\to\infty} \omega(t) = \omega_d. \tag{8}$$

The goal of velocity servos is to ensure velocity regulation of their embedded operational shaft and body attached to it.

The PI controller is a common inner controller used in velocity servo actuators owing to its simplicity in design and tuning. From an automatic control point of view, the velocity servo actuators internal block structure is depicted in Figure 4, their command input signal is denoted by u which traditionally stand for the desired velocity ω_d of the shaft velocity ω and their output is ω, being τ_d an external mechanical load torque disturbance.

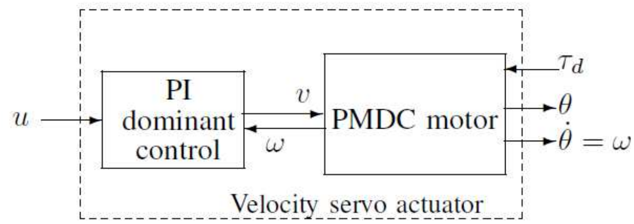

Figure 4: Velocity servo internal structure block diagram: Dominant PI control of PMDC motors.

Many low cost robotic apparatuses such as desktop robot arms and wheeled mobile robots intended for entertainment and educational objectives, are equipped with velocity servo actuators whose parameters have been tuned by the servos manufacturer (these parameters are fixed, so they cannot be modified).

With reference to Figure 4, in an analog velocity servo actuator the embedded inner analog PI controller is modeled by:

$$v = k_P\widetilde{\omega} + k_I\xi \tag{9}$$
$$\dot{\xi} = \widetilde{\omega}, \tag{10}$$

where

$$\widetilde{\omega} \triangleq u - \omega, \tag{11}$$

being u the velocity servo input signal (typically in an open–loop fashion: the velocity setpoint ω_d), and where the positive constants: $k_P > 0, k_I > 0$ are the PI control parameters or gains (proportional and integral gains respectively).

The closed–loop system describing the PI control of PMDC motors is obtained by substituting the control action —armature voltage motor input— v from control law (9) into the motor model (5):

$$JR\dot{\omega} + (kk_b + Rb_0)\,\omega = k\underbrace{\left[k_P\widetilde{\omega} + k_I\xi\right]}_{v} - R\tau_d,$$

whose time derivative yields:

$$JR\ddot{\omega} + [(kk_b + Rb_0)]\,\dot{\omega} = k\left[k_P\dot{\widetilde{\omega}} + k_I\dot{\xi}\right] - R\dot{\tau}_d. \tag{12}$$

By invoking definitions in (10) and (11), and rearranging, equation (12) becomes:

$$JR\ddot{\omega} + [(kk_b + Rb_0) + k\ k_P]\,\dot{\omega} + k\cdot k_I\omega = k\,[k_P\dot{u} + k_I\ u] - R\dot{\tau}_d, \tag{13}$$

which leads to the velocity servo model:

$$\begin{aligned} JR\ddot{\omega} + [(kk_b + Rb_0 + k\cdot k_P)]\,\dot{\omega} + k\cdot k_I\omega &= \\ = k\ k_P\ \dot{u} + k\ k_I\ \ u - R\dot{\tau}_d. \end{aligned} \tag{14}$$

For the sake of compact notation one defines the following a_is positive constants:

$$\left\{ \begin{array}{ll} a_0 \triangleq JR; & a_1 \triangleq [(kk_b + Rb_0 + k\cdot k_P)]\,; \\ a_2 \triangleq k\cdot k_I; & a_3 \triangleq k\ k_P. \end{array} \right\} \tag{15}$$

There is not assumption about the positiveness of above a_is parameters in (15) but as a matter of fact, they are positive by the way they were defined (products and additions of positive constants).

Thus, model (14) is rewritten as:

$$a_0\ddot{\omega} + a_1\dot{\omega} + a_2\,\omega = a_3\dot{u} + a_2 u - R\dot{\tau}_d. \tag{16}$$

This may be seen as a two inputs (u and τ_d) and one output (ω) linear system whose transfer function is:

$$\omega(s) = G_D(s)(a_3\; s + a_2)\mathcal{U}(s) - R\; G_D(s) s \mathcal{T}_d(s), \tag{17}$$

where

$$G_D(s) \triangleq \frac{1}{[a_0 s^2 + a_1 s + a_2]}. \tag{18}$$

Since a_is are positive constants, hence the zeroes of its characteristic polynomial have negative real part. This shows that the PI control of PMDC motors is stable and for disturbance free ($\tau_d \equiv 0$) situations also the velocity servos modeled by (16) (for constant $u = \omega_d$) achieve speed regulation ($\omega(t) \to u = \omega_d$ as $t \to \infty$). As a matter or fact, this worthy conclusion is also valid for any constant disturbing torque τ_d.

3.1. A simplified dynamic realistic velocity servo model

In summary, based upon the velocity servo actuator block diagram in Figure 4 and its associated closed–loop system (16), this paper proposes the following realistic dynamic velocity servo model:

$$a_0\ddot{\omega} + a_1\dot{\omega} + a_2\,\omega = a_3\dot{u} + a_2 u - R\dot{\tau}_d, \tag{19}$$

This model is effective as far as the following assumptions hold:

A1. Negligible armature inductance L of the inner brushed PMDC motor.

A2. Inner, either PI control or PI–dominant PID control.

A3. No gearbox reduction.

With regard to assumption A3, it can be easily relaxed without loosing neither the servo model (19) structure nor properties nor qualitative servo behavior conclusions.

3.2. Qualitative behavior

Below, one looks at the analysis of the velocity servo system (19) behavior via its transfer function (17) assuming constant control action $u = \omega_d \in \mathbb{R}$ and without mechanical load disturbance: $\tau_d \equiv 0$.

Under disturbance free situation $\tau_d = 0$, and because ω_d was assumed to be constant, then from (17) and final value theorem one gets:

$$\lim_{t\to\infty} \omega(t) = \left([G_D(s)[a_3\; s + a_2]] \Big|_{s=0} \right) \cdot \omega_d = \omega_d,$$

as desired in velocity regulation control objective (8). For this reason (good velocity regulation effectiveness: $\omega \to \omega_d$ in absence of torque disturbance), velocity servo actuators are widely utilized as driving elements into a number of robotics setups such as mobile wheeled robots and drons).

4. Conclusions

This paper has introduced a dynamic model for velocity servo actuators composed by armature-controlled PMDC motors under analog PI feedback control. They are able to achieve global speed regulation under disturbance free applications. The model is described by a nonautonomous linear second order system which allows to confirm the following observed behavior in laboratory experimental tests:

- Velocity servo actuators are effective for speed regulation when applied to operate mechanisms under negligible mechanical torque loads or in special mechanisms such as planar horizontal Cartesian robots (x–y printer like mechanisms). In such cases, the traditional open–loop control is effective to achieve the user desired speed or positioning goal.
- Notwithstanding, velocity servo actuators may fail handling mechanisms under presence of strong general torque or forces disturbances such as coupled multi-body mechanisms under gravity action: e.g., general 3D robot arms, or humanoids —biped robots—, as well as wheeled mobile robots or propelled based drones. In order to enhance the system performance, further feeback outer-loop control may be required.

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