

Proposal for the nomenclature of reaction wheel arrays Propuesta para la nomenclatura de arreglos de ruedas de reacción

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Abstract

Reaction Wheel Arrays (RWA) are groups of two or more actuators, known as reaction wheels, which operate together to achieve the desired attitude control of an aircraft, space vehicle or satellite. The RWA are exceptionally useful actuator configurations, and have been the subject of extensive studies in the last decades. However, despite these efforts, many areas of opportunity persist in the present. In this work, a proposed solution is addressed and developed for one of them: the absence of a general and well-defined mathematical rule to refer in a precise and reliable way to the different types of RWA that exist in the state of the art.

Keywords: attitude, satellite, reaction wheel, RWA, actuator.

Resumen

Los Arreglos de Ruedas de Reacción (RWA, por sus siglas en inglés), son agrupaciones de dos o más actuadores, a su vez denominados ruedas de reacción, que operan en conjunto para alcanzar el control de actitud deseado de una aeronave, vehículo espacial o satélite. Los RWA son configuraciones de actuadores excepcionalmente útiles, y han sido objeto de cuantiosos estudios en las últimas décadas. Sin embargo, a pesar de dichos esfuerzos, es un hecho que persisten muchas áreas de oportunidad en la actualidad. En este trabajo, se aborda y desarrolla una propuesta de solución para una de ellas: la ausencia de una regla matemática general y bien definida para referirse de una manera precisa y confiable a los diferentes tipos de RWA del estado del arte.

Palabras Clave: actitud, satélite, rueda de reacción, RWA, actuador.

1. Introduction

The Reaction Wheel (RW) is an extremely versatile, powerful and useful actuator. Its applications go from inverted pendulums to control of orientation – hereinafter “attitude” – of aircraft, spacecrafts, airplanes, drones, planes, CanSats and satellites (Cortés-García, 2020); especially for CubeSat, thanks to their compact size and clean operation (California-Polytechnic-State-University, 2022).

In general, the use of RW is growing due to their reliability, durability, performance, robustness and more properties that make them highly desirable, (Oland and Schlanbusch, 2009).

The RW are active actuators powered by electric energy, which is converted into mechanic power by a direct current electric motor (Lechuga-Gerónimo et al., 2021). The output power is expressed in angular speed and torque, which then is amplified by an inertia disk on the motor shaft (Sugita, 2017).

In figure 1 there is an illustration of a reaction wheel, centered in an arbitrary frame.

In practice, and specially in aerospace applications, it's extremely common to find the RW in sets called Reaction Wheel Arrays (RWA) (Ismail and Varatharajoo, 2010), (Oland and Schlanbusch, 2009). By this way, they enhance their individual capabilities to meet a specific goal. An example of previous statement is found in figure 2, where it is shown an Gyroscope Moment Control (GMC) composed by a RWA of 4 RW, manufactured by Airbus for the aircraft attitude control (Airbus, nd).

While arrays have been extensively studied for properties such as performance (Dae-Kwan, 2014), driver compatibility, efficiency and effectiveness (Shirazi and Mirshams, 2014) (Sidi, 1997) (King, 2020); however, there is not a precise way to refer to every RWA existent in the state-of-art; also, if a tilt angle is included, it becomes a sophisticated issue (Yoon, 2021).

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The absence of a universal manner to refer to every RWA represents an enormous problem because it may lead into confusions and misinterpretation while working with a big number of these actuators in one single RWA, which increases if there are multiple RWA with several RW. Therefore, in this proposal, an effort is made to solve the issue.

In order to present the solution, this work is structured as follows: in section 3, it is presented an analysis of the state of art related to RWA properties such: types, distribution and more, the RWA matrix. In section 4, the nomenclature's problem becomes exposed. In section 5, a nomenclature proposal is reached. Finally, in section 6, some examples are provided to demonstrate the effectiveness of the proposal.

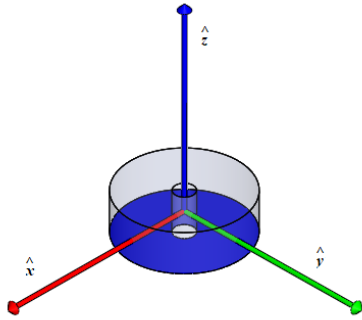


Figure 1: Illustration of an RW over an arbitrary frame and centered at origin.



Figure 2: Control Moment Gyroscope (GMC), model 15-45 S, made by Airbus. (Airbus, nd)

2. Notation and Abbreviations

The notation and abbreviations used along this work are established as follows.

2.1. Notation

- A : Matrix.
- b : Vector.
- \hat{b} : Unitary vector.
- r : constant.

2.2. Abbreviations and Acronyms

- RWA: Reaction Wheel Array.
- RW: Reaction Wheel.

3. Reaction Wheel Array (RWA)

In this section, the RWA state-of-art is studied. Although they have several characteristics, the most important are three: 1) properties, 2) types and 3) matrix; all of them presented in next subsections.

3.1. RWA Properties

1. **Number.** The quantity of RW in the array.
2. **Position.** The location of the RW with respect to a given frame of reference.
3. **Distribution.** The geometry that resembles the RWA with respect to the position of the RW.
4. **Redundancy.** It refers to the capacity of one or more RW are able to cover each other in case of failure.
5. **Colinearity.** Unlike the property of dynamical systems, it refers to whether an RW is coincident with one of the principal axes of the reference frame.
6. **Optimization.** It indicates if a wheel is capable of reaching a certain degree of inclination (*tilt angle*), from which energy consumption can be optimized.

3.2. Types of Reaction Wheel Arrays

There is a certain number of well-defined RWA. However, the reality is that there infinite number due to the infinite combinations that are possible.

Recently, in (Lechuga-Gerónimo, 2023), an effort was made to classify and name them according to these properties, where it is concluded that five are the most important, plus a sixth in this work. The proposal consists of a binomial structure given by the rule in equation (1) to refer to each RWA.

$$A - B \quad (1)$$

where A is the number of RWs that builds the array, while B is the one of the six type of distribution presented below.

- **P: Perpendicular.** When two wheels are perpendicular to each other, considering that both are collinear with two different main axes.
- **O: Orthogonal.** Every RW is exactly colinear to each axis.
- **TR: Tetraedral Regular.** Similar to the above, but with a redundant RW on one – and only one – of the axes in the opposite direction.
- **TI: Tetraedral Irregular.** Similar to previous, but with the two RW redundant with an inclination.
- **PYR: Pyramidal.** An improved version of previous, where all the wheels are tilted by an angle.

- **C: Cubic.** Similar to Ortogonal, but with an additional RW in opposite direction for each axis.

Two examples are given as follows.

- **(2-P),** equals to 2 Reaction Wheel in perpendicular configuration. See part a) in figure 3.
- **(3-O),** equals to 3 Reaction Wheel in orthogonal configuration. See part b) in figure 3.

3.3. Most Common Reaction Wheels Array

The popularity of one RWA depends on how useful it is. For example, there are RWA that provides partial attitude control very useful for certain applications such CanSats. By the other hand, there are RWA with a big number of RW that offers complete attitude control and redundancy in all axis. In this subsection, a list of the most relevant RWA is provided.

1. Two Reaction Wheels in Perpendicular (2-P).

It only has two RW and they are coincident with two of the three main axes. It is incapable of offering complete attitude control.

The 2-P is particularly useful for cases where there is not enough space (i.e. CanSats) (Çelebi et al., 2011), by sufficiency when only partial control is desired, or when there is already another actuator to cover the remaining axis. This array is illustrated in part a) of figure 3.

2. Three Reaction Wheels in Orthogonal (3-O).

It is the simplest RWA to offer complete attitude control. It's comprised of three collinear RW to each axis. This RWA is extremely useful for small CubeSats close to 1 Unity, where there is not so much space to allocate for the attitude control system.

An illustration can be seen in part b) of figure 3.

3. Four Reaction Wheels in Regular Tetraedal (4-TR).

Similar to 3-O, this RWA counts with an additional and collinear RW in one axis, which allows the capability to generate twice the angular momentum on it. For this reason, 4-TR is very useful for those satellites that have a significantly larger axis of inertia.

An illustration is shown in part c) of figure 3.

4. Four Reaction Wheels in Irregular Tetraedal (4-TI).

Similar to previous case, but with the difference that the two redundant RW are tilted by an angle in relation to the horizontal plane of the reference frame, which causes the RWA to become partially redundant.

An illustration can be seen in part d) of figure 3.

5. Four Reaction Wheels in Pyramidal (4-PYR).

This RWA is made up of four RW inclined by a tilt angle and whose distribution resembles a pyramid; where it gets its name from. This RWA is considered the most challenging to implement, but so are the benefits.

On the one hand, there is better energy efficiency because the RW rotates at a lower speed since the generated torque is divided among the four. On the other hand, there is also superior redundancy because it offers the ability to continue operating in case one RWA fails, although a higher energy cost. Despite this, the 4-PYR remains as one of the favorites RWA.

An illustration can be seen in part e) of figure 3.

6. Six Reaction Wheels in Cubic (6-C)

It is a fully-redundant RWA, similar to 3-O, in which there are two RWs for each axis of inertia, although one for each direction.

An illustration can be seen in part f) of figure 3.

This classification performs really well at the time of giving name to diverse RWA as an early approach. However, it is not universally valid, as it is studied in next section.

The last part of this section is about a mathematical representation for arrays, known as the RWA Matrix.

3.4. The RWA Matrix

The RWA Matrix is a convenient and compact mathematical representation for storing information about an RWA (Ismail and Varatharajoo, 2010), specifically their number and distribution.

This is accomplished by a matrix expression, $\mathbf{A}_R \in \mathbb{R}^{n \times 3}$, where n implies the number of RWs, and whose arguments represent their vector components.

\mathbf{A}_R is also defined as a matrix composed of n unit vectors of the n wheels of the RWA. For a better illustration, see the equation (2) and its expansion in equation (3), which in both cases. $\hat{\omega}_n \in \mathbb{R}^{3 \times 1}$ represents the angular velocity vector of the n th wheel RWA reaction.

$$\mathbf{A}_R = [\hat{\omega}_1 \quad \hat{\omega}_2 \quad \dots \quad \hat{\omega}_{n-1} \quad \hat{\omega}_n] \quad (2)$$

$$\mathbf{A}_R = \begin{bmatrix} \hat{\omega}_{1_x} & \hat{\omega}_{2_x} & \dots & \hat{\omega}_{n-1_x} & \hat{\omega}_{n_x} \\ \hat{\omega}_{1_y} & \hat{\omega}_{2_y} & \dots & \hat{\omega}_{n-1_y} & \hat{\omega}_{n_y} \\ \hat{\omega}_{1_z} & \hat{\omega}_{2_z} & \dots & \hat{\omega}_{n-1_z} & \hat{\omega}_{n_z} \end{bmatrix} \quad (3)$$

The RWA Matrix is very useful at the time to calculate the influence of torque generated by every RW in the array to the satellite or aircraft. In next section, the central issue of this work is explored and a solution is offered as well.

4. The Nomenclature's Problem

The problem with the previous classification is that RW location in a RWA is unknown. For example, in 2-P, it is clear that two RW composes the 2-P array and that they should be in perpendicular position and must be coincident with two axes. However, it is not clear in which axes they are coincident. It is open to free interpretation if the RW are located over \hat{x} and \hat{y} , or \hat{y} and \hat{z} ; or even maybe in $-\hat{x}$ and $-\hat{y}$.

In other RWA, such 3-O, the problem is not so obvious, since the three RWs are necessarily coincident with the principal axes, which reduces to an identification problem (i.e. to know *which* RW is *which* one). The same problem occurs for 6-C, and similarly for 4-PYR. However, for higher order cases, the problem increases.

For example, in 4-TR it is known that there would be four RWs, but not if the wheel A is collinear with the \hat{x} , \hat{y} or \hat{z} axis, which is further complicated in 4-TI array.

Also, there is the possibility of having an additional sub-type of array, named as *incongruent*, assigned to those RWA

that do not fully coincide with the axis because of the existence of a tilt angle. This can be seen in figure 4 from the study in (Ismail and Varatharajoo, 2010), where the array cases from 2 to 11 were wrongly handled as similar to those which – in the opposite –, fully coincide with the axis; from now denominated as *congruent*.

For the so-called *incongruent* cases, it is evident that the system proposed in the previous section is not completely adequate, so this represent a new nomenclature's challenge.

In summary, the three above problems are resumed and identified as:

1. **Identification.** The capability to know exactly which (not *where*) RW is in any RWA.
2. **Position.** The capability to determine in which axis a certain RW is on; considering the tilt angle of inclination and direction.
3. **Congruence.** The property of RWA whose distribution is consistent due to variations in tilt angle.

In the next section, a solution is explored.

5. Proposal Creation

Several properties and characteristics of the RWA has been listed. It is clear now that the RWA are sets of reaction wheels, which are very useful for diverse applications with focus in aerospace industry. The absence of an standard nomenclature in literature to name certain RWA composed of n reaction wheels is worrying, since may lead into confusions and misinterpretations, especially when working with a big number of wheels.

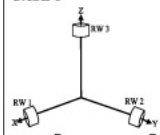
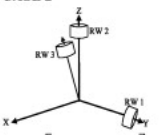
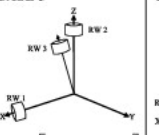
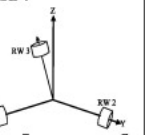
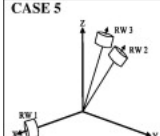
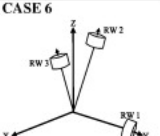
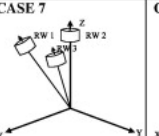
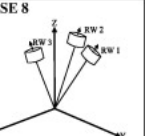
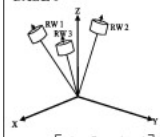
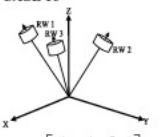
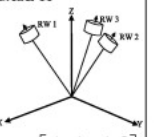
CASE 1  $A_w = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	CASE 2  $A_w = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ <p>-RW 3 tilted on (x, y) plane.</p>	CASE 3  $A_w = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ <p>-RW 3 tilted on (x, y) plane.</p>	CASE 4  $A_w = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ <p>-RW 3 tilted on (x, y) plane.</p>
CASE 5  $A_w = \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ <p>-RW 2 tilted on (-x, y) plane. -RW 3 tilted on (-x, y) plane.</p>	CASE 6  $A_w = \begin{bmatrix} 0 & 0 & -1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ <p>-RW 2 tilted on (-x, y) plane. -RW 3 tilted on (x, y) plane.</p>	CASE 7  $A_w = \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$ <p>-RW 1 tilted on (x, y) plane. -RW 3 tilted on (x, y) plane.</p>	CASE 8  $A_w = \begin{bmatrix} 0 & -1 & -1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ <p>-RW 1 tilted on (-x, y) plane. -RW 2 tilted on (-x, y) plane. -RW 3 tilted on (x, y) plane.</p>
CASE 9  $A_w = \begin{bmatrix} 1 & 0 & -1 & 1 \\ -1 & 0 & -1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$ <p>-RW 1 tilted on (x, y) plane. -RW 2 tilted on (-x, y) plane. -RW 3 tilted on (x, y) plane.</p>	CASE 10  $A_w = \begin{bmatrix} 1 & -1 & 0 & 1 \\ -1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$ <p>-RW 1 tilted on (x, y) plane. -RW 2 tilted on (-x, y) plane. -RW 3 tilted on (x, y) plane.</p>	CASE 11  $A_w = \begin{bmatrix} 1 & -1 & -1 & 0 \\ -1 & 1 & -1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ <p>-RW 1 tilted on (x, y) plane. -RW 2 tilted on (-x, y) plane. -RW 3 tilted on (-x, y) plane.</p>	

Figure 4: Some *incongruent* RWA. (Ismail and Varatharajoo, 2010)

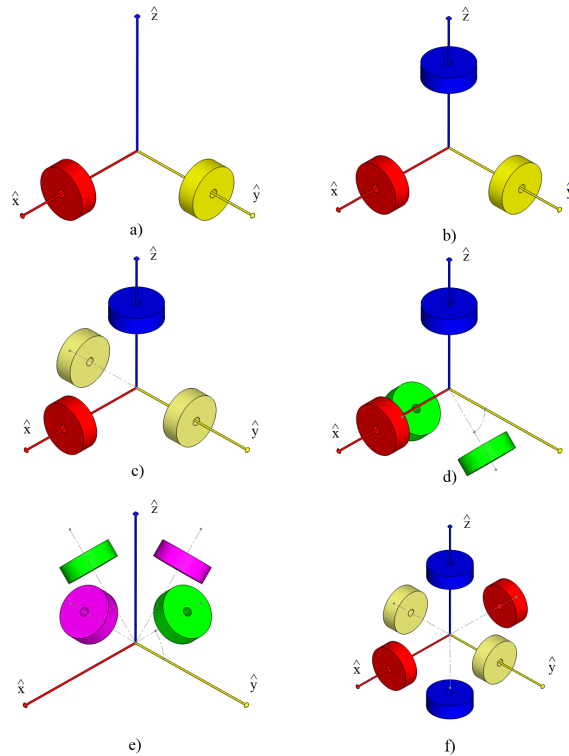


Figure 3: Types of Reaction Wheel Arrays: a) 2-P, b) 3-O, c) 4-TR, d) 4-TI, e) 4-PYR, f) 6-C.

Despite there is the RWA Matrix as a tool for calculating the total torque generation of the RW, at this moment is not useful for such purpose.

With all of these statements, the proposal consists on adopt and use the RWA Matrix as a basis for the nomenclature, which can be adapted to truly meet the goal to provide an easier and absolutely clear interpretation of RWA, solving the three problem resumed in the end of previous section.

To do so, five rules are proposed as follows.

5.1. Rules for Nomenclature Proposal

These steps receive a short name and a related description. All the steps are consecutives.

1. Identification.

The RWs are named starting from the axis in which they are located.

- First, all RWs coincident with the principal axes are considered, in the order given by \hat{x} , \hat{y} and \hat{z} .
- Secondly, in positive sense: $+\hat{x}$, $+\hat{y}$ and $+\hat{z}$; in that order.
- And finally, those RW in the negative sense ($-\hat{x}$, $-\hat{y}$ and $-\hat{z}$), in the same sequence as previously.

The intention is that they occupy a clearly identifiable place within the RWA Matrix, and the allocation of places within it is not arbitrary, as commonly occurred in literature.

2. Denomination.

A proper name is assigned to each of the RWs, which is given in alphabetical order, denoted by Latin letters (i.e. a , b , c ...). The purpose of this is to have a reliable way to name them in which there is absolutely no confusion when referring to a specific RW.

In the RWA Matrix, the order is given by a consecutive structure. This means the first RW is called a , the second one is b , and so on, until letter z ; just like in equation (4).

$$\mathbf{A}_R = \begin{bmatrix} a & b & c & \cdots & z \end{bmatrix} \quad (4)$$

Further cases of a RWA containing RW beyond the letter z are not contemplated, since RWAs with more than 26 RW are, unlike, probable.

It is important to remember that, according to the definition in subsection 3.4 and more specifically in equation (2), the RWA Matrix contains the angular velocity vectors of every RW in the array. Therefore, the letters in equation (4) are substituted by vectors with the corresponding notation, as established in equation (5).

$$\mathbf{A}_R = \begin{bmatrix} \omega_a & \omega_b & \omega_c & \cdots & \omega_z \end{bmatrix} \quad (5)$$

3. Unitary

In previous works, such as (Ismail and Varatharajoo, 2010), non-unit vectors have been used in the RWA matrix. This may lead to a misinterpretation of the vector direction.

Because of this, the use of exclusively unit vectors is recommended, since these are the minimum and most

compact expression to represent the position and distribution of the RWA according to a reference frame. In consequence to this statement, the previous equation (5) is transformed into the equation (6).

$$\mathbf{A}_R = \begin{bmatrix} \hat{\omega}_a & \hat{\omega}_b & \hat{\omega}_c & \cdots & \hat{\omega}_z \end{bmatrix} \quad (6)$$

Until now, the result is pretty much similar to the RWA Matrix. However, two rules more are needed in order to correctly satisfy the problem.

4. Subscripts and Superscripts.

With the elements up to now, there are sufficiency conditions to clearly identify an RWA. Nevertheless, it is possible to propose an improvement.

In the event that it is required to quickly visualize an arrangement by its RWA Matrix without stopping to review the vector components, what may be done is to take advantage of the space available in the subscripts and superscripts, placing the RW number first; while for the second, the RWA to which they belong.

For example, see equation (7). It is clear now that it's a 3-O array by just looking the matrix and without having to analyse the vectorial components.

$$\mathbf{A}_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_3^O \quad (7)$$

5. Notation for incongruents.

The incongruent cases are – as discussed before – special, since they resemble the congruent ones in distribution, but despite the similarities they are not identical and should not be treated as such.

For this reason, to distinguish them from the rest, the notation of a tilde (\sim) will be used whenever it refers to one of them.

For example, the 2-P array, in which there is an RW with a certain degree of inclination, will be called $2\text{-}\tilde{P}$. This should be enough to quickly recognize that this is not exactly the same RWA, despite of sharing a same basis.

As a manner of illustration for the application of these rules, the next examples are provided.

5.2. Examples for Nomenclature Proposal

Twelve scenarios are proposed, the first six for congruent RWAs and the rest for incongruent. Every case has its RWA Matrix along with a figure for better comprehension of the proposal's rules.

Congruents

Case 1: 2-P

In this case, the RW are located in the \hat{x} and \hat{y} axes. The RWA Matrix is expressed in equation (8), and its illustration in figure 5.

$$\mathbf{A}_2^P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}_2^P \quad (8)$$

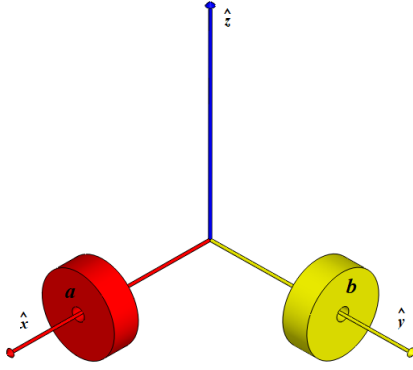


Figure 5: Case 1 illustration.

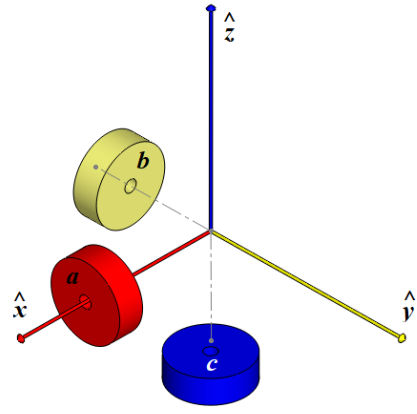


Figure 7: Case 3 illustration.

Case 2: Alternative 2-P

For this case, let us suppose that one RW is located in $-\hat{x}$ and the other in \hat{z} . The RWA Matrix is found in equation (9), and the illustration in figure 6.

$$\mathbf{A}_2^P = [\hat{\omega}_a \quad \hat{\omega}_b]_3^O \begin{bmatrix} 0 & -1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}_2^P \quad (9)$$

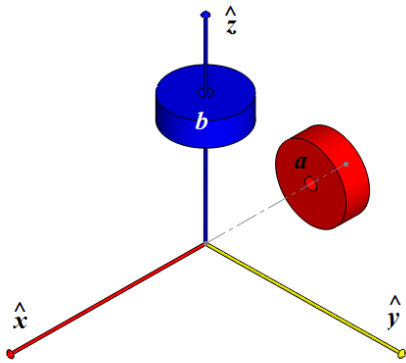


Figure 6: Case 2 illustration.

Case 3: Non-conventional 3-O

In this case, a different three orthogonal RWA is selected. The RWA Matrix is in equation (10) and illustration in figure 7.

$$\mathbf{A}_3^O = [\hat{\omega}_a \quad \hat{\omega}_b \quad \hat{\omega}_c]_3^O = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}_3^O \quad (10)$$

Case 4: 6-C

This case offers a RW in all three axes and positive and negative directions as well. It's excellent to show the effectiveness of this proposal. The RWA Matrix is found in equation (11) and illustrated in figure 8.

$$\mathbf{A}_6^C = [\hat{\omega}_a \quad \hat{\omega}_b \quad \hat{\omega}_c \quad \hat{\omega}_d \quad \hat{\omega}_e \quad \hat{\omega}_f]_6^C \quad (11)$$

$$\mathbf{A}_6^C = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix}_6^C$$

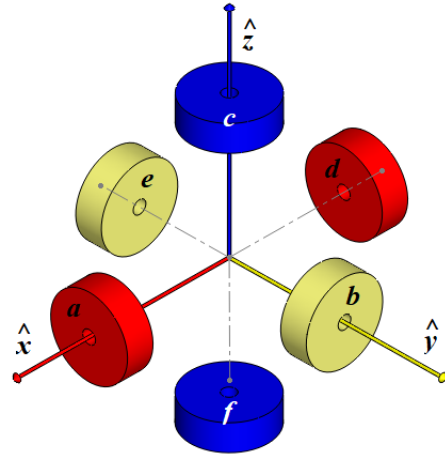


Figure 8: Case 4 illustration.

Incongruent

For these special cases, four scenarios are proposed, all of them with a tilt angle ($\beta \in \mathbb{R}$) in one or more RW.

Case 5: 2-P

In this case, there are two RW in perpendicular, but one on them is tilted between \hat{y} and $-\hat{z}$; as described in equation (12) and visualized in figure 9.

$$\mathbf{A}_2^{\tilde{P}} = [\hat{\omega}_a \quad \hat{\omega}_b]_2^{\tilde{P}} = \begin{bmatrix} 1 & 0 \\ 0 & \sin(\beta) \\ 0 & -\cos(\beta) \end{bmatrix}_2^{\tilde{P}} \quad (12)$$

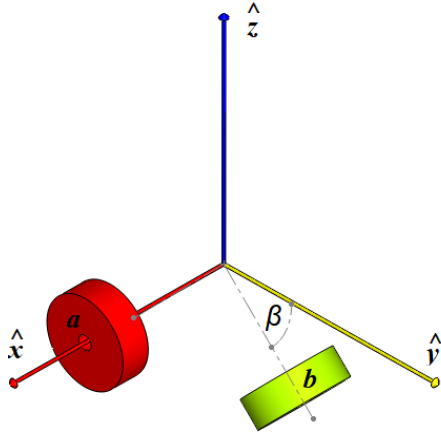


Figure 9: Case 5 illustration.

Case 6: 3- \tilde{O}

For this case, a classic 3 RW in Orthogonal, there is one RW that has a tilt angle \hat{y} and $-\hat{z}$. The RWA Matrix is expressed in equation (13) and illustrated in figure 10.

$$\mathbf{A}_3^{\tilde{O}} = \begin{bmatrix} \hat{\omega}_a & \hat{\omega}_b & \hat{\omega}_c \end{bmatrix}_3^{\tilde{O}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin(\beta) & 0 \\ 0 & -\cos(\beta) & 1 \end{bmatrix}_3^{\tilde{O}} \quad (13)$$

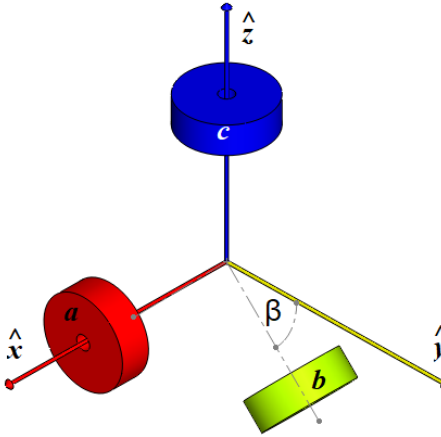


Figure 10: Case 6 illustration.

Case 7: 4- $P\tilde{Y}R$

In this last case, it is about a Pyramidal set but with two different tilt angle. Also, one of the RW has null inclination (is collinear with $-\hat{y}$). The RWA is described in equation (14) and illustrated in figure 11.

$$\mathbf{A}_4^{P\tilde{Y}R} = \begin{bmatrix} \sin(\beta_1) & 0 & -1 & 0 \\ 0 & \sin(\beta_2) & 0 & -\sin(\beta_2) \\ \cos(\beta_1) & \cos(\beta_2) & 0 & \cos(\beta_2) \end{bmatrix}_4^{P\tilde{Y}R} \quad (14)$$

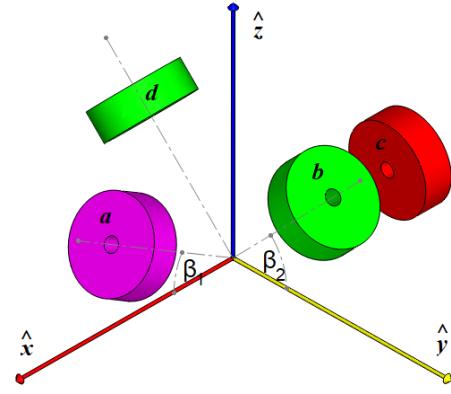


Figure 11: Case 7 illustration.

With this last scenario, the illustration of how this proposal works is considered finished. Some final comments and recommendations may be found in next and final section.

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Conclusions

The current proposal offers a clear, simple, optimized and unambiguous representation in the way the reaction wheels and their arrays are identified. It has allowed to name different RWA in a unique way, avoiding misinterpretations about where are the RW inside the array; and even more, providing a name to them (a, b, c,...), which is unrepeatable and compact as well. It should be remarked the fact that, for the very first time in history of RWA, an ordered sequence for assign an identification was offered.

These key factors are very important at the moment of working with several actuators, i.e. in case 4, where there are six or more RW in one single array.

The benefits of this proposal are evident at the time of working from the most simple RWA to those more sophisticated.

Having such a reliable method is important since reaction wheels are favorite actuators and will continue to be used for aerospace science and technology development either by academic and industry sectors. Because of this, it's time to adopt a unique manner to denominate the diverse RWA that are possible to find in all the state-of-art.

Also, it is important to clarify that, although this method is applicable for reaction wheels, in principle, it is all compatible with other actuators or their assemblies, like magnetic dampers (Rodríguez-Torres et al., 2022), or in the rocket control systems (Gómez-León et al., 2023).

For example, in the use of the so-called *magnetorquers*; where one or more electromagnetic coils are used for attitude control through interaction with the Earth's magnetic field; or even innovative actuators such spherical reaction wheel (Dae-Kwan et al., 2014).

The applicability remains as a pending topic for research and possible future work.

In the end, is crucial to remember that is only one proposal. It is possible that in the incoming time another method may appear, with even more benefits for the RWA nomenclature.

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