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Intruder penetration in a vibrated and sheared two-dimensional granular medium

Penetración de un intruso en un medio granular con vibración y esfuerzo cortante

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Abstract

In this work, a passive intruder is moving through a tilted quasi-two-dimensional vibrated and sheared granular medium of soft spherical particles. Driven by the gravitational forces produced because of the slight inclination of the system, the passive intruder moves and penetrates throughout the granular medium. The dynamics of the intruder and the dynamical and structural characteristics of the particles in the medium are both determined. For obtaining the dynamical properties, the mean squared displacement of the intruder and each particle in the medium were estimated. The structural changes in the media, which are the outcome of the motion of the intruder, were observed and analyzed. The radial distribution function and the six-fold bond orientational order parameter were used to structurally characterize the medium, both with and without intruder penetration. Based on the different experiments performed, it was found that the intruder's trajectories fall into regions of lesser order in the granular medium.

Keywords: Intruder penetration, granular medium of soft particles, structural changes.

Resumen

Se estudia el movimiento de un intruso pasivo mientras penetra en un medio granular bidimensional de partículas suaves, sometido a vibración y a esfuerzo cortante. Debido a la ligera inclinación del sistema, el intruso pasivo se mueve y penetra a través del medio granular. Tanto la dinámica del intruso como las características dinámicas y estructurales del medio granular fueron determinadas y son reportadas a continuación. Se calculó el desplazamiento cuadrático medio del intruso y de cada partícula granular. También se observaron y analizaron los cambios estructurales en el medio granular, los que son resultado del movimiento del intruso y los que son producidos por la geometría con que fue aplicado el esfuerzo cortante en el medio. Para la caracterización estructural se utilizaron la función radial de distribución y el parámetro de orden orientacional, ante la presencia del intruso y sin ella. Con base en los diferentes experimentos realizados, se encontró que las trayectorias seguidas por el intruso coinciden con las regiones de menor orden en el medio granular.

Palabras Clave: Penetración de un intruso, medio granular de partículas suaves, cambios estructurales.

1. Introduction

Motivated by its ubiquitous presence in nature, the phenomena that involve granular materials have been studied for a long time (Jaeger et al., 1956)(Hisao et al., 1995)(P. Richard, 2005). A granular material is defined as a collection of rigid macroscopic particles, whose size allows them to remain unaffected by thermal fluctuations (O. P. B. Andreotti, 2013). Recent research has expanded the classical definition of granular materials to include deformable or soft particles in certain experimental contexts, allowing for a broader understanding of both thermal and non-thermal behaviors in granular systems. For example, (Barés, 2022) introduced the concept of "squishy granular matter," referring to grains capable of significant shape changes without breaking, thus enriching the study of how deformable particles interact and respond under various conditions. (Vescovi, 2016) examined shear flows of deformable particles and developed constitutive relations that describe the transition between fluid-like and solid-like states in granular media. Collectively, these studies

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underscore the importance of incorporating particle deformability to capture the complex behaviors of granular materials. Complementing this approach, recent experimental studies have explored the thermal-like behaviors that granular systems can exhibit under specific conditions. For instance, (Sakaï et al., 2017) demonstrated an equation of state in dense granular suspensions, showing a relationship between granular pressure and packing fraction reminiscent of thermal systems. (Merminod et al., 2014) observed a transition from dissipative to quasi-elastic behavior in a 2D system of magnetized particles as magnetic repulsion increased. (Shattuck, 2009) observed distinct gas, dense gas, and crystal states in quasi-2D granular materials, with structures resembling equilibrium hard disk simulations. (Song, 2005) measured an effective temperature in jammed granular materials using a sheared system of spherical beads, with probe particles equilibrating at the same temperature regardless of their characteristics. Together, these studies highlight that, although granular materials typically operate outside thermal equilibrium, they can exhibit properties analogous to equilibrium systems when specific conditions are met, particularly in systems with deformable particles. These materials are an essential part of our daily lives, they are involved in many industrial processes and several studies have been developed around granular phenomena, but in this work, the penetration of a passive intruder into the granular medium is of particular interest. Studying an intruder moving throughout a granular medium has presented several scientific inquiries and applications. Passive intruders that impact and move through 3D and quasi-2D granular systems have been explored, both numerical and experimentally, to determine the viscous damping forces (M. Hou, 2005), the depth dependence on the friction forces that act normally to the intruder surface (T. A. Brzinski, 2013), and the critical force (F_c) that define the transition between different range of motions (Silbert, 2013), among many other investigations (Ruiz-Suárez, 2013) (Mehdi Omidvar, 2014). Cooperative interactions between multiple intruders penetrating a granular medium have been observed in a vertical 2D granular medium of polystyrene slight spheres (F Pacheco, 2010), resembling the collective behavior that appears in living systems. Even though some attempts have been made (González-Gutiérrez, Jorge, 2018), the structural changes that appear in granular media as the intruder penetrates have not yet been fully understood. As these previous investigations have been made in an experimental situation more related to the intruders falling through the air, is reasonable to believe that with the addition of a soft and flexible granular medium that forms stabler structures and opposes a greater resistance to the intruder motion, the complexity of the phenomena would be enriched and the structural changes produced will be easier to quantify. Some research has been made in this regard (Aymeric Merceron, 2019) (Chen, 2023), but a study at a particle level of the structural changes in the granular medium had not been achieved for a passive intruder penetration.

Since passive intruders cannot move independently throughout the granular medium, an external force must act on them to move, and on the granular medium to fluidize it. Vibration has been used (J. B. Knight, 1993) (D. A. Huerta, 2005) to fluidize granular media by continuously injecting energy to maintain the kinetic energy of the granular particles. However, other methods are also useful to change the granular medium, such as mechanical deformations. The last ones could be applied to the system by periodically deforming the particle container. Previous work using both vibration and mechanical deformations (F. López-González, 2022) studied the structural properties of a granular medium that depends on vibration and shearing with the coexistence of regions with different degrees of order in the medium. The structural changes produced in the granular medium were explored, also at the particle level, for an active intruder penetration (C. G. Vidales-Hernández, 2024).

The main purpose of this work is to identify a relation between the trajectory followed by the intruders and the degree of structural order in the granular medium. It is reasonable to expect that when a passive intruder falls because of the action of gravity through the granular medium, its diffusion will be affected by the strength or brittleness of the structures formed in the granular medium. The intruder would presumably move faster in regions with losing packing and remain stuck for some time in regions with high structural order degree. Since it is a passive intruder, the direction and persistence of its movement will depend on the external forces that may act on it, and this effect would be reflected in their trajectories. In comparison, an active intruder, which has an internal motor that provides it with a directed motion, performs less erratic trajectories. Another phenomenon that may present differences between the penetration of a passive intruder and an active intruder is the structural changes that occur at short distances around its position. Previous work, developed under similar experimental conditions with an active intruder shows that at short range around an active intruder, there are no important structural modifications. Exploring the case of passive intruder penetration would be helpful in determining the effect of the activity in the structural modifications in the vicinity of intruders and could shed light on the different motion behaviors between active and passive matter.

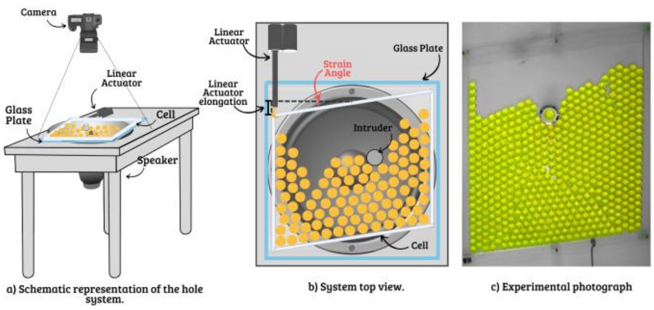


Figure 1. Experimental setup. (a) Schematic representation of the system where the different parts could be seen. The camera records from above and the container is formed by a glass plate and a deformable frame. The table where everything is supported could be inclined by making two of their legs smaller. (b) System's top view, the effect of the linear actuator on the cell frame could be seen. The shear aperture is a measure of the deformation of the cell.

2. Methodology

2.1. Experimental setup

The experiments consisted of a passive intruder. The intruder is a cylindrical laboratory weight made of steel, weighing 100 g. Its dimensions are 2.0 cm in diameter and 1.5

cm in height. This intruder moves throughout a wet and soft granular medium due to the action of external forces such as gravity, given the slight inclination of the system, and vibration. The granular medium comprises 500 soft polyacrylic acid particles of 1.0cm diameter, which form a two-dimensional bed contained in a deformable cell formed by a glass plate with a rectangular frame of 26.0cm in width and 29.0cm in height. The deformable frame consists of four edges with pivots at their vertices that allow the rotation of the connected edges as well as the translation of the vertices. Additionally, there are two fixed pivots, which only generate rotation, located at the midpoint of two parallel edges. This configuration allows for the formation of rhomboids that, in their minimal deformation, form a rectangle. The shearing is produced by the antiparallel movement of the edges without fixed pivots, generating one-dimensional shear. The container deformation is reached by the action of a 12V linear actuator connected to one of the corners of the frame. The frequency and amplitude of the shearing are adjusted by a Labview homemade program that controls the voltage and frequency of the signal in the linear actuator, providing a periodical deformation of the container. In this work, five repetitions of the experiment were conducted for each value of the aperture of the shear stress. This is measured in terms of the distance between the original and final positions of the frame corner (see Figure 1). The values chosen for the maximum aperture of the deformation were 2.0, 1.5, 1.0, and 0.5 cm, and the shear frequency was fixed at 0.1Hz. The shear frequency of 0.1 Hz was chosen to enable the study of the granular medium's structural response at low frequencies, allowing the gravitationally driven intruder to interact with particles in a way that minimizes high-frequency vibrations, which could obscure the gradual, order-reducing structural dynamics around the intruder. This frequency also aligns with established research (F. López-González, 2022; 2023) in vibrated granular systems, supporting comparisons in structural characterization methods like the radial distribution function and bond orientational order parameter.

Another fluidization mechanism is vibration, which was provided by a speaker fed with a sinusoidal signal ($A\, sin\, sin\, 2\pi f t$) on which the glass plate is supported. To quantify the system vibration, we used the Γ parameter, which is obtained by:

$$\Gamma = \frac{4A\,\pi^2 f^2}{g} \quad (1)$$

where A and f were the amplitude and frequency of vibration, which in this case were 0.1cm and 90Hz., respectively. Also, Γ is a measure of the ration between the system and gravity accelerations. These experiments were performed for $\Gamma = 3.55$.

Also, the entire system is slightly tilted (3°) to drive the particles to the bottom of the container, so the granular medium and the passive intruder are subject to a component of the gravitational force.

We ensured sufficient energy to prevent granular collapse due to dissipation by carefully tuning the vibration parameters of the system. The quasi-two-dimensional setup allowed for optimal energy distribution, while the vibrational frequency and amplitude were chosen to sustain particle movement and prevent clustering. This configuration minimized dissipation-induced collapse, maintaining a stable granular medium for the passive intruder to traverse. Additionally, the inclined setup assisted in redistributing particles, providing a balance against gravitational compaction and ensuring continuous flow and particle rearrangement around the intruder. The values used are similar to those in reference (F. López-González, 2022), where the transition from an amorphous to crystalline phase in a slightly tilted, bidimensional vibrated hard-sphere system under periodic shearing was studied, effectively countering dissipation effects in granular media.

To obtain the system's dynamics, each experiment is recorded from above, with a standard resolution of 720 x 540px at 30 fps, videos length ranges from 1 to 5 minutes. This time is determined by the time it takes for the intruder to reach the bottom of the cell. The image sequences were digitally processed using a specialized image processing software (ImageJ) (Schneider, Rasband, & Eliceiri, 2012). There the trajectories for the intruder and each granular particle in the medium were obtained to analyze their dynamics by determining their mean square displacements (MSDs). The radial distribution functions (RDF) were also determined with this software. And the data obtained from the nearest neighbors, using the Delanuay criterion (Delanuay, 1934), was used to define the sixth bond-orientational order parameter (Ψ'_6). This parameter enables us to identify the local effect of the intruder on the structure formed by the surrounding granular particles.

2.2. Methods

To study the dynamics of the intruder and of each particle in the medium, the mean square displacement was used. It is a measurement of the deviation of the position r(t) as a function of time taking as reference the initial position r(0).

$$\langle \Delta r^2(t) \rangle = \langle |r(t) - r(0)| \rangle^2 \quad (2)$$

The solution of the Langevin equation gives the temporal evolution of the motion:

$$\langle r^2(t) \rangle = \frac{6\,k_B T \tau^2}{m} \left[\frac{t}{\tau} - \left(1 - e^{-\frac{t}{\tau}} \right) \right] \quad (3)$$

There, two regimens could be observed. When the characteristic time (τ) is bigger than the variable time $\tau \gg t$ then the mean square displacement obeys a quadratic relation that corresponds to ballistic motion. When $\tau \ll t$ the behavior is Brownian-type, and the time follows the relation:

$$\langle r^2(t) \rangle = 2d\, D_E t \quad (4)$$

Where D_E is the Einstein's diffusion coefficient and d denotes the dimension where diffusion motion takes place. By means of a linear fit in the MSD curves, the diffusion coefficient could be determined. But even when the motion is not Brownian, MSD is still a useful tool to describe the dynamics of particles. To determine the regimen of motion, a power law could be fitted $\langle r^2(t) \rangle = A\, t^B$, then one can stablish if a motion is super-diffusive, diffusive, or sub-diffusive based on the value of the exponent in time. A periodical behavior of motion could be also observed in MSD curves.

To globally determining the structural characteristics of the granular medium the Radial Distribution Function (RDF) is used, and locally is the sixth-fold bond-orientational order parameter (Ψ'_6) that give us the structural information. RDF gives the probability density of finding other particles at a distance (r) from the particle taken as reference and it is denoted as g(r). For the calculation, the image processing software ImageJ (Schneider, 2012) was used, which employs the method known as "Guard Area." This method consists of defining a central region of reference particles, called "Guard Area", to prevent the maximum radial distance of the annular profiles from exceeding the boundaries of the sample. The complement of this central region, with respect to the surface where the sample lies, contains particles that will be included in the annular count but will not be considered as reference particles. In ImageJ, the central quadrant is taken by dividing the rectangular image into a 3x3 matrix of quadrants. This calculation method is fast because it only averages a part of the sample, but it excludes information near the boundary, making it unsuitable for small samples. This is a particularly important tool for studying fluids, but it has also been used in granular materials as it is useful to describe the structure of systems and to
discriminate between different phases of matter. The global information that is provided from the g(r) is obtained by averaging over a sufficiently large sample with a considerable number of particles, in this sense it does not provide information about the local structure of aggregates in the system. That is the purpose of the parameter Ψ'_6, that is employed when the particles spheres and tend to form hexagonal close-packed structures in 2D. This parameter is a measure of the local order around each particle in the granular medium, and compares the position of its neighbors with the hexagonal structure, this could be seen in the following relation:

$$\Psi_6 = \frac{1}{n_i} \left| \sum_{j=1}^{n_i} e^{i6\theta_{ij}} \right| \qquad (5)$$

Ψ_6 is a function of the number of neighbors n_i, given by the Delaunay triangulation, and of the angle θ_{ij} that is formed between the line that joins the reference particle i with its neighbor j and the x axis. A new condition is introduced to this parameter to consider only the number of neighbors that are bounded n_b, that is made by taking a cut-off distance as 1.1σ, where σ is the particle's diameter. This new parameter gives better information about the real structure of the granular medium.

3. **Results**

3.1 Dynamical analysis

Experiments were performed five times for each shear deformation value from the mentioned above and five times when the granular medium is subject only to vibration. Here the intruders are falling under the action of gravity and because of the external perturbations, so in principle they are trying to follow a straight line to the bottom of the cell. However, their collisions

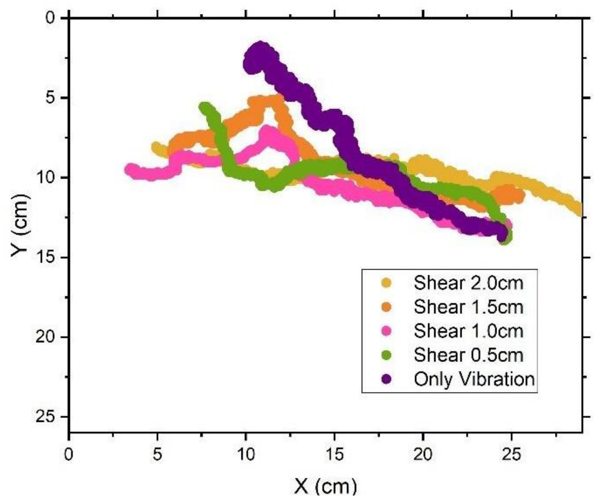


Figure 2.Trajectories followed by the intruders, each color represents an experiment with different conditions of deformation, with only vibration and without intruder.

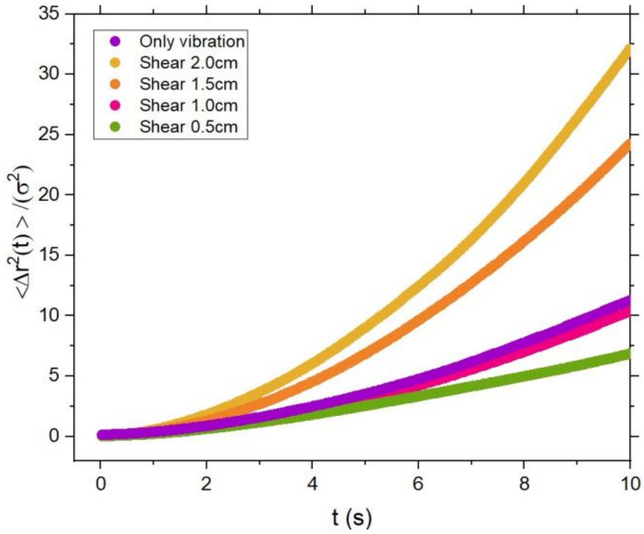


Figure 3. Mean square displacements of the intruder as it penetrates the granular medium, each color represents an experiment with different conditions of deformation, with only vibration and without intruder.

with the granular medium introduce noise into their trajectories. The goal from this work is to find a relation between the structure of the granular medium and the trajectories followed by the intruders.

A graph with the trajectories of the passive intruder could be found in figure 2. The intruders start almost at the same point and goes almost to the bottom of the cell when shear stress is applied, and to one side of the cell when only vibration is present. In all cases the trajectories are shown deflected when they reach a certain region near the bottom of the cell.

The dynamical features were obtained from the calculation of the MSDs and taking an average with other repetitions. Granular particles exhibit a super-diffusive motion for short times when subject to shear stress with exponent values that range between 1.51 and 1.66. In figure 3, it is observed that the MSD curves grow with the deformation of the cell. The periodicity of the deformation produces periodic curves for the MSD. An experiment where there is not intruder penetration was also performed and the MSD reached smaller values than for the case with intruder penetration. Meaning that the intruder also produce motion into the granular medium. When only vibration is applied to the granular medium, the MSD no longer shows a periodical behavior. Instead, it is almost a constant pretty near to zero with only small fluctuations, and its motion for short times is diffusive

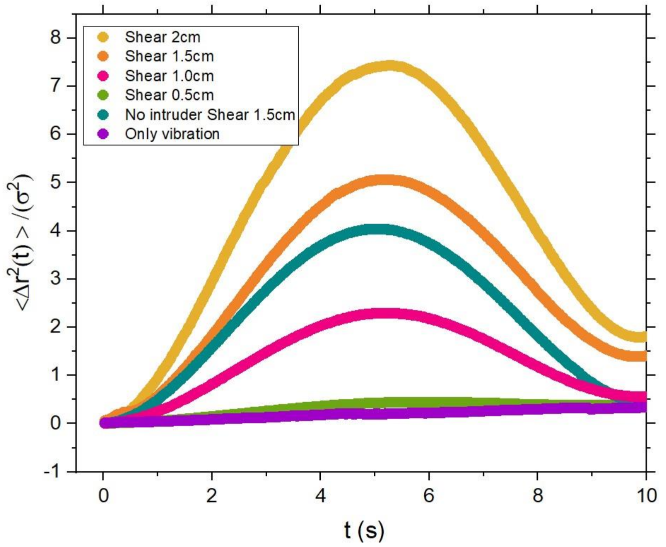


Figure 4. Mean square displacements of the granular medium, each color represents an experiment with different conditions of deformation, with only vibration and without intruder.

with exponent in time equals to 1.03. In this case, the granular particles are not traveling a great distance, but they are on average kept vibrating around some equilibrium positions. From this, it is concluded that the vibration itself is not sufficient, for this system, to fluidize the granular medium.

The mean square displacements of the passive intruders were also calculated. Figure 4, show that the displacement of the intruder grows monotonically with time, with no periodic behavior as compared with the granular medium, where the periodicity of the shear strain controls most of its motion. Instead, the passive intruder falls through the bottom, only deviating when the granular structure puts up the most resistance. By applying the power law fit to this MSD curves, it is found that the motion belongs to super-diffusive regimen and that both, the exponent in time and the maximum values reached by MSD, grows as function of the shear stress applied.

A similar behavior is observed in the MSD when shear stress is not applied to the system, with the only difference being that their motion through the granular medium occurs slower than in most of cases when shear was applied. However, the exponent in time for the passive intruder motion when only vibration is applied is almost the same than for the smallest value of deformation. Having also, a super-diffusive type of motion.

3.2 Structural analysis

To globally describe the structure of the granular medium, the radial distribution function was used. This function was calculated for an illustrative video of each value of deformation and for the cases with only vibration and without intruder. In every one of them, the characteristic shape of the hexagonal close-packed structure is (figure 5). It makes sense because this structure achieves the highest concentration of spheres in two dimensions. Nevertheless, the local structure remains hidden in this quantity.

To visualize the local order around each granular particle in the medium, the sixfold bond-orientational order parameter (Ψ'_6) were determined. The value obtained for this parameter is associated with a color scale being blue the higher values of Ψ'_6,

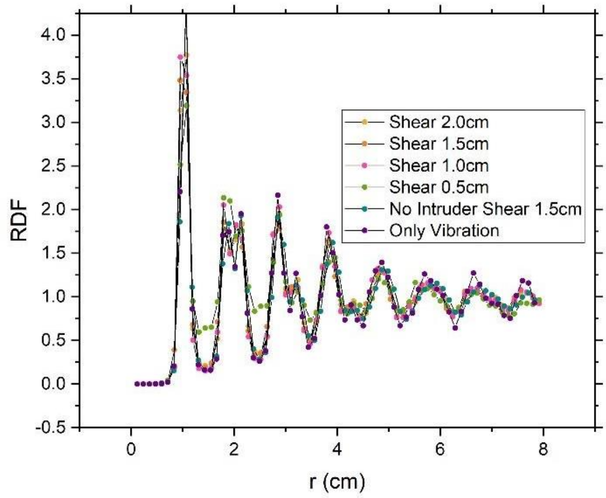
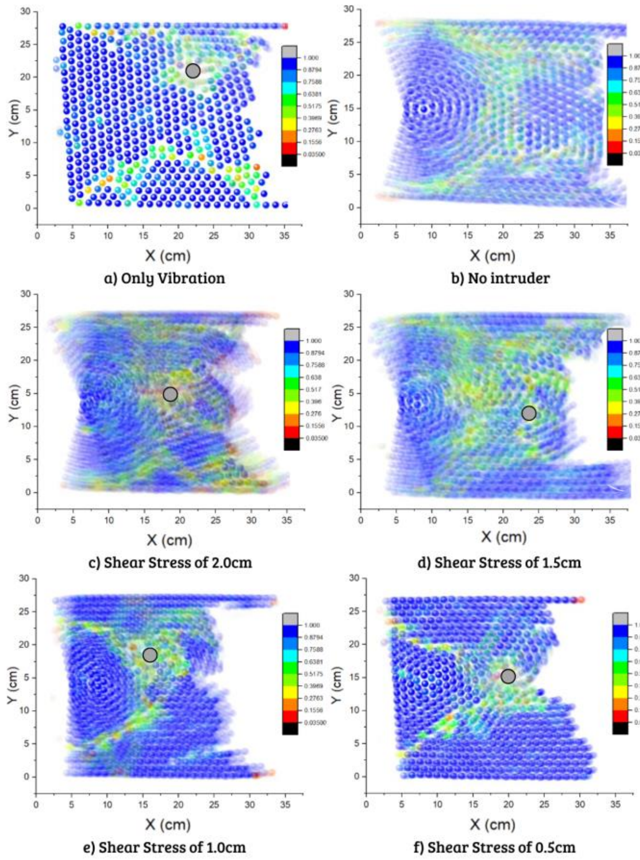


Figure 5. Radial distribution functions (g(r)) of the granular particles, each color represents an experiment with different conditions of deformation, with only vibration and without intruder.



meaning higher order, and red the lowest values of Ψ'_6, meaning less order. Each particle in the system is depicted in graphs associated with this color scale, and by making a z-projection of 250 frames of the video, a graphical average of the structure in time could be obtained. And a comparison between this average structure for each of the experimental conditions is seen in figure 6. The combined effect of vibrations, shear stress and slight inclination of the system makes the particles sediment at the bottom of the cell. Producing a circular blue region at the bottom that is associated with higher structural order. These regions kept their structural order even after the cyclic deformations of the system. Right up this region, a colorful half-round can be observed, denoting a region with less structural order, which occurs at lesser or greater extent depending on the amount of deformation of the cell, and which does not appear when shear stress is not applied. This indicates that the emergence of this regions is directly related to the geometry and intensity of the shear stress applied.

Since the case without an intruder was carried out with a 1.5cm of shear deformation, the effect of the intruder on the granular medium could be observed when Figures 6-b and 6-d are compared. Figure 6-b shows that the intruder is moving across the central part of the cell. In the surrounding region of the intruder, less structural order denoted by the more generalized colorful region is observed. On the other hand, figure 6-d shows a structure with high order in the central part, when no intruder is present, the only disorder that could be observed is the one produced by the geometry of the shear stress that was applied.

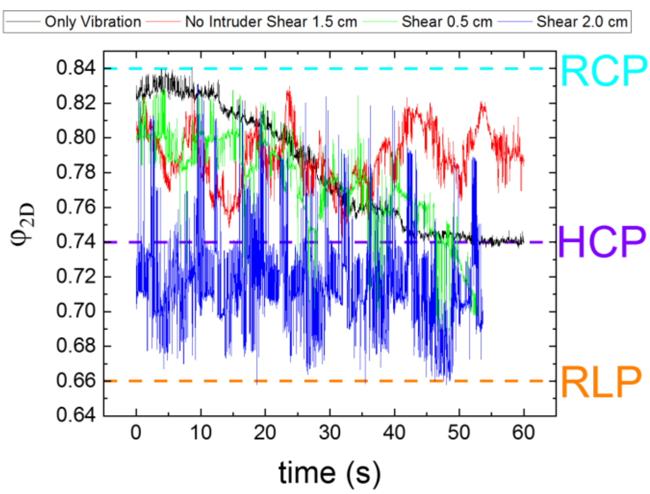


Figure 7. Temporal evolution of the 2D planar density φ_{2D} for the hydrogel sphere system under representative conditions: vibration without the intruder and shear (black), in the absence of the intruder with vibration and a shear amplitude of 1.5 cm (red), in the presence of the intruder with vibration and shear amplitudes of 0.5 cm (green) and 2.0 cm (blue), respectively. The dashed lines correspond to characteristic planar density values for Hexagonal Close Packing (HCP, purple), Random Close Packing (RCP, cyan), and Random Loose Packing (RLP, orange) for sphere granular systems. These benchmarks serve as references for identifying structural transitions in the system.

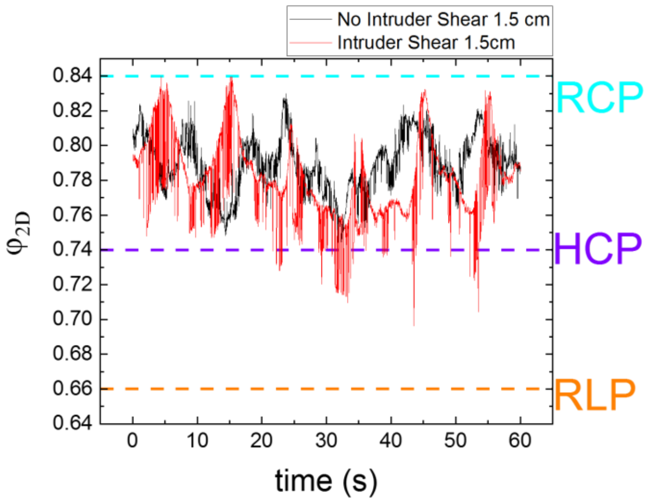


Figure 8 Temporal evolution of the 2D planar density φ_{2D} for the hydrogel sphere system in the absence of the intruder with vibration and a shear amplitude of 1.5 cm (black) and in the presence of the intruder with vibration at the same maximum shear amplitude (red). The dashed lines correspond to characteristic planar density values for Hexagonal Close Packing (HCP, purple), Random Close Packing (RCP, cyan), and Random Loose Packing (RLP, orange) for sphere granular systems. These benchmarks serve as references for identifying structural transitions in the system.

Additionally, the temporal evolution of the 2D planar density, φ_{2D}, was calculated using the convex hull criterion to define the bulk area, as used in the reference (F. López-González., 2023). As demonstrated in previous studies (Reis, 2006), (Jin, 2014) (Brouwers, 2023), the Random Close Packing (RCP) in two-dimensional sphere systems has a density of $\varphi_{2D}\approx 0.85$, while Hexagonal Close Packing (HCP) and Random Loose Packing (RLP) is around $\varphi_{2D}\approx 0.74$ and $\varphi_{2D}\approx 0.67$, respectively. The denser configuration corresponds to RCP, where particles become more compacted. In figure 7, the temporal evolution of the 2D planar density is shown for the representative cases previously presented in figure 5. As observed, subjecting the sample solely to vibration and inclination drives the granular sample to values close to hexagonal ordering (black curve). The application of cyclic shear with an amplitude of 1.5 cm (red curve) accelerates this hexagonal ordering. However, due to the high shear amplitude, the system does not achieve a monocrystalline state. These results align with previous investigations (Yu, 2006; F. López-González, 2022). When the intruder is added, we can see that for small shear amplitudes (green curve), the planar density decays to values near Hexagonal Close Packing (HCP) approximately 30 seconds earlier than in the vibration-only case. For high shear amplitudes (blue curve), the decay is more pronounced, and the planar density oscillates over a wide range of values, transitioning from near Random Loose Packing to values slightly below Random Close Packing, with an average close to HCP. To delve deeper into the effect of the intruder on the compaction of the sample, figure 8 shows the temporal evolution of the 2D planar density with and without the intruder (red and black curves, respectively) under identical shear conditions. Here, the graph of planar density vs. time for a sheared medium with an amplitude of 1.5 cm in the presence of the intruder commences with values slightly below as if there were no intruder, and reaches values of planar density close to HCP at the middle of the experiment. At this time, the intruder was positioned around the center of the granular medium. This suggests that the impact of the intruder is more pronounced when it is centered within the medium. As the intruder moves beyond the geometric center of the sample, the planar density tends to return to the values observed in the absence of the intruder. This indicates that the intruder plays a key role in influencing the compaction process. The behavior is driven by the intruder's effect, which compacts the spheres and pushes them towards the walls, forming small clusters. The splitting of peaks observed in g(r) reflects this transition, indicating that the particle arrangement evolves into more ordered configurations. Specifically, the splitting of the second peak is attributed to changes in local particle compaction, rather than other granular phenomena, such as granular collapse.

4. **Conclusions**

Describing the relationship between the structural changes in the granular media and a passive intruder motion was the main objective of this work. For this porpoise, the experiments performed consisted of a passive intruder that moves through a vibrated, shared and slightly tilted two-dimensional granular system. The observations indicate that the shear amplitude of deformation is in direct relation to the intruder's dynamics. As the amplitude of deformation increases, the intruder is able to displace easier through the granular medium and it could reach the bottom faster. This is explained because the shear acts on the system by rearranging the granular medium producing regions with less structural order. Since vibration cannot increase over a certain value without losing the system´s two-dimensional character, it is observed that the dynamics of the intruder is slowed down for the case when no shear is applied.

Additionally, it was clear that the collisions with particles in the granular bed prevented the intruder from following a straight- line trajectory. It is more difficult for the intruder to traverse the regions at the bottom of the cell where granular medium is more structured compared with the lateral regions where the geometry of the shear applied produces less structural order. This sedimentation of granular particles that occurs at the bottom is because of the slight inclination of the system and favors the appearance of hexagonal close-packed structures that dominates the global description provided by the radial distribution function. From a comparison between figures 6-b and 6-d, it could be concluded that the intruder has a short-range effect in the granular medium structure. This effect is not produced by vibrations or mechanical deformations because it does not appear under the same experimental conditions but without an intruder.

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