Robust network control to time delays on a robot manipulator
Control robusto en red a tiempos de retardo para un brazo manipulador

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Resumen
Sistemas de control retroalimentado con una red inalámbrica como canal de información entre los sensores y el controlador y los actuadores se les conoce como Sistemas de Control en Red (NCS). El proceso de análisis y diseño de un controlador apropiado para un NCS, convierte en algo muy diferente la aplicación de los métodos tradicionales. Sistema de control retroalimentado en plantas con múltiples entradas y múltiples salidas con canales de comunicación inalámbricas presentan problemas tales como competencia por los recursos de la red, pérdidas de paquetes de datos y presencia de tiempos de retardo. Este trabajo se enfoca en mejorar la respuesta de un sistema retroalimentado con retardos en sus trayectorias. Varias metodologías de diseño de controladores han sido propuestas para tratar el problema de retardos inducidos, dependiendo del tipo de retardo i.e., retardos constantes, variables continuamente, discretamente o aleatoriamente y en sistemas MIMO retardos independientes en cada canal. Este trabajo trata con el diseño de un controlador H∞ robusto en red, para un NCS con tiempos de retardo acotados dentro de un intervalo tal que el sistema en bucle cerrado logre satisfacer las especificaciones de funcionamiento para valores amplios de los tiempos de retardo. El método se aplica a un robot de tres grados de libertad.

Palabras Clave: Sistemas de Control en Red, tiempos de retardo, control robusto.

Abstract
Feedback control systems with network as information channels between sensors and between controllers and actuators, are called Network Control Systems (NCS). Network in the loop makes the analysis and design of a proper controller quite different from the traditional ones. Controllers for MIMO plants with wireless communication channels have different type of problems as to compete for the network resources, losses of data packages and presence of communication delays. This work deals with the problem of delays present in the feedback system. Various methodologies have been proposed to deal with the problem of network induced delays which depends on the nature of the delay as: constant, continuous discrete and random delays, and in MIMO systems independent channel delays. This paper deals with design of a robust H∞ controller for a MIMO system with time delays limited to an interval, such as obtain a close loop system satisfy robust stability and robust performance specifications in ample ranges of time delays. The method is applied to a three degree of freedom arm manipulator.

Keywords: Network Control Systems, time delays, robust control.

1. Introduction

Feedback control systems with wireless communication channels between sensors and controllers, and controllers and actuators are called Network Control Systems NCS. Applications of NCS can be found in the nuclear industry where a robot needs to operate under high levels of radiation as is explained in [Jackson R.,1984], and [Larcombe M.H.E. et al. 1984] or in hostile environments as in the chemical, steel, and mining industries that limit the use of electronic hardware also in bilateral telecontrol [Leung, G.M. et al. 1997], and [Cavusoglu C. M. et al. 2001]. In a multiple-input multiple-output MIMO network control system, sensors nodes as well as controllers compete for the network resources at the same time, resulting in losses of data packages and presence of delays. Networks in the loop made
the analysis and design of a proper robust controllers more difficult since they introduce phase lagging at the operating frequency band that may cause degradation of performance or even take the system to instability. Various methodologies have been proposed to deal with performance and stability conditions for a MIMO NCS see for example [Fu, M. et al, 1993], and [Verriest,E.I. et al. 1991] that have worked in the problem of robust stabilization of system with delays looking for extending the stability and performance of the controlled, to ample values of the time delay. This paper deals with the problem of improving the performance of a NCS by keeping the system under function specifications for an ample range of the time delays. The time delays are assumed to be upper bounded. The Hoo optimization controller design method has proved to be an effective method to improve the system performance when there exist model and parameter uncertainty in the system; see [Gonzalez L. et al. 2000], and [Gonzalez L. et al. 2003]. Here, by taking the time delay as part of a multiplicative uncertainty in the plant it may be possible to design an Hoo controller for the worst-case to obtain a NCS that gives good performance in an ample values of the time delays. By a simple operation on the NCS transforms the close loop system with delays in the direct and feedback paths into a close loops system with one double delay in the direct path and a delay at the output of the feedback system. This allows to take the delay in the loop as an uncertainty in the model of the plant. Then a robust controller Hoo design is carried out in order to obtain a feedback robust controller that extends the performance of the system for longer time delays an example is given on a three degrees of freedom arm manipulator with network induced but bounded time delays. Comparisons were made with respect to a classical PI controller and the difference on performance of the system between the two methods is noticeable. The paper is divided as follows; in Section I the model of the system with a double time delay in the direct path of the loop is obtained. The time delays are now taken as a multiplicative input uncertainty of the plant. In Section II is described the process on the system to carried out the Hoo design process. The procedure to define the weighting transfer matrices, and the conditions that must be satisfied in order to arrive to a robust performance and robust stable controlled system. Finally, conclusions and directions of research are presented in Section III.

2. Multivariable Modeling of the Time Delay for purpose of design

The structure of the NCS considered for this case is shown in Fig.1.

The robot was taken as three inputs, three outputs multivariable system with matrix transfer function representation $G(s)$ for which a local PI regulator $K(s)$ has previously been designed and applied to form the regulated three by three system $M(s)$. The output vector is composed of the three angular positions of the arms of the robot. The direct and feedback multivariable communication channels were modeled as diagonal matrices as:

$$
\begin{bmatrix}
e^{-\tau_1} & 0 & 0 \\
0 & e^{-\tau_2} & 0 \\
0 & 0 & e^{-\tau_3}
\end{bmatrix}
$$

and

$$
\begin{bmatrix}
e^{-\tau_4} & 0 & 0 \\
0 & e^{-\tau_5} & 0 \\
0 & 0 & e^{-\tau_6}
\end{bmatrix}
$$

The six time delays were taken as unknown, independent, but upper bounded, that is $\tau_i < \tau_{\max}$ where $\tau_{\max}$ is the maximum value allowed for delay $\tau_i$ for $i=1,2,\ldots,6$. In order to facilitate the analysis and design, the time delays were bounded by a single value $\tau_{\max}$ given by $\tau_{\max} = \max\{\tau_{\max}\}$ for $i = 1,2,\ldots,6$. Applying simple operations in the close loop system, an equivalent system more appropriate for the approach of taken the time delays as an uncertainty of the plant, was derived and is shown in Fig. 2. In this new structure time delays appear only in the direct path of the feedback control system and at the output of the close loop. The time delays in the direct path is now twice the value of the previous structure and will be taken as a multiplicative uncertainty at the output of the nominal plant $M(s)$. $K_c(s)$ is the multivariable controller to be designed.

Fig. 2: Equivalent system.

The output sensitivity matrix function $S_p(s)$ of the system in the close loop that relates the vector measured output $y_{meas}(t)$ with the disturbances at the output of the plant $M(s)$ is given by:

$$
S_p(s) = \left(I + e^{-2\tau M(s)K_c(s)}\right)^{-1}
$$

It is clear from the above expression how the time delays enter and affect the frequency response and stability of the close loop system by increasing the rate of the phase Another useful expression in the design of a robust controller is the complementary sensitivity matrix function $T(s) = I - S_p(s)$ given as:

$$
T(s) = \left(I + e^{-2\tau M(s)K_c(s)}\right)^{-1} e^{-2\tau M(s)K_c(s)}
$$

The output of the system to a reference command is obtained as:

$$
y_{ref}(s) = T(s)r_{ref}(s)
$$

indicating that the actual output of the system will experience a $\tau$, secs. direct delay with respect to a command signal. Generally speaking, random delays are present in all communication channels with magnitudes depending on the characteristics of the channel. Stochastic analysis has been the general tool used for analysis of this class of systems.
Here, instead the Hoo norm optimization method based on the “worst case” policy was applied. While the method is notice for producing conservative controllers, this can be seen as an advantage for security reasons.

The actual plant $M_a(s)$ is taken to have model uncertainty that in this case is taken as input multiplicative uncertainty at the input of the nominal plant $M_n(s)$ as shown in Eq. (4).

$$M_a(s) = M_n(s)\left(i + \Delta(s)\right)$$

where $\Delta(s)$ is a non-structured multiplicative uncertainty upper bounded., that is $\|\Delta(j\omega)\|_{\infty} < \beta$ for $\beta > 0$. In block diagram Eq. 4 is represented as in Fig.3

![Fig.3: Multiplicative uncertainty modeling.](image)

Now from Fig. 2, taking the actual plant $M_a(s)$ as $e^{-2\pi T_m s} M_n(s)$ in the NCS and making equal to Eq. 4,

$$M_n(s)(e^{-2\pi T_m s}) = M_n(s)(i + \Delta(s))$$

From Eq.5 a non-structured multiplicative uncertainty model as function of the time delays may be derived as:

$$\Delta(s) = e^{-2\pi T_m s} I - I$$

Eq. 6 shows how the parametric uncertainty of the time delay $T_m$ has been incorporated in the multiplicative uncertainty $\Delta(s)$. Hence, a class of uncertainty $\Delta_m$ may be defined such that for every time delay $T < T_m$ there is a $\Delta(s) \in \Delta_m$ and $\beta > 0$ such that $\|\Delta(j\omega)\|_{\infty} < \beta$ and is given by:

$$\|\Delta(j\omega)\|_{\infty} = \max_{\omega} |e^{-2\pi jT_m \omega} - 1| < \beta$$

Fig. 4 shows the frequency response curves for different values of the time delay of the multiplicative uncertainty model of Eq. 6.

![Fig. 4: Frequency response of the multiplicative uncertainty $\Delta(j\omega)$.](image)

3. Hoo Design Process

3.1. Outline of the Hoo design process

The objective of the design is to compute a robust controller for an NCS, by only using output feedback such that the controlled system remains stable on ample values of the time delay and also fulfill the following design specifications.

- Tracking of a step reference signal with a steady state error less than 1%
- Sensor and communication channels noise attenuation.
- No actuator saturation.

In order to apply Hoo optimization design method to derive a robust controller to satisfy the above specifications the augmented system shown in Fig. 5 is proposed.

![Fig. 5: Augmented system.](image)

Here the $W_i(j\omega)$ i=1, 2...4 are 3x3 weighting matrices transfer functions to be designed. $W_o(j\omega)$ is designed to normalize the multiplicative uncertainty as:

$$\|W_o(j\omega)\|_{\infty} < 1$$

$W_i(j\omega)$ models or normalizes the control signals amplitude under different performance conditions such that actuators saturation be avoided. $W_n(j\omega)$ emulates sensor noise, and $W_e(j\omega)$ gives as output the vector signal ze(t) that indicates the system performance when subject to reference, and disturbance inputs. The external signals to the system are: the reference and noise signals, r(t), and n(t), and wo (t) that represents the manner the uncertainty affects the system. y(t) is the input to the controller that for this case is the error signal of the system, u(t) is the control signal, and $y_m(t)$ is the measured output signal for purpose of feedback. The output signals to measure the performance of feedback control system are $z_u(t)$ the normalized control signal, $z_e(t)$ the normalized error signal, and finally za(t) the equivalent weighting response to model uncertainty. From a simple analysis of the diagram of Fig.5 the following transfer function matrix between the external vector signal $\vec{w} = [w_r^T \ R^T \ \eta^T]^T$ and the output vector signal $\vec{z} = [z_u^T \ z_e^T \ z_a^T]^T$ of the augmented system is derived as:

$$\Gamma_{wz} = \begin{bmatrix} -W_o S_o K_e & W_o S_o K_e & -W_o S_o K_e W_n \\ -W_o S_o M & W_o S_o & W_o S_o W_n \\ -W_o S_o K_e M & W_o S_o K_e & -W_o S_o K_e W_n \end{bmatrix}$$

Hence, the design Hoo norm optimization problem is stated as follows:
Compute a suboptimal admissible controller $K_c(j\omega)$ such that the close loop system became robust stable for ample values of the communication channels time delays, and with robust performance with respect to design specifications, such that for a given $\gamma > 0$ it is satisfied the following inequality:

$$\min_{C(s)} \| \Gamma_{wz} \|_{\infty} \leq \gamma$$ (9)

### 3.2. Weighting matrices selection

Briefly in this section is described how the weighting matrices were obtained for design purpose.

The weighting matrix $W_o(j\omega)$ is chosen such that the inverse of its norm for all frequencies covers the family of frequency responses of the multiplicative uncertainty corresponding to different values of the time delay in order to satisfy Eq. (8).

Based in the frequency responses of Fig. 4, after some trials an uncertainty weighting transfer function that satisfies the above condition was computed as:

$$W_o(j\omega) = \frac{22387(s + 0.409 \times 10^{-3}) + 2}{(s + 0.4)(s + 1)}$$ (10)

which frequency response is also shown in Fig. 4 and clearly covers the frequency responses of Eq.7. $W_n(j\omega)$ as the noise weighting function, models the frequency spectrum of the noise present in the system. It was found that the servo-potentiometer, used for position sensing, were the source of high intensity noise at low frequencies as compared with the communication channel noise, so, a high-pass filter model was proposed with 35 dBs attenuation up to two octaves from the bandwidth of the nominal plant ($\omega_{ab} = 0.42$ rad/s), and with a 6 dBs constant attenuation at high frequencies. So, the corresponding weighting transfer matrix derived was:

$$W_n(s) = \frac{2(s + 4)}{s + 80}I_3$$ (12)

$$W_p(s) = \frac{2s + 4}{s + 80}I_3$$ (11)

The control weighting matrix $W_u(j\omega)$ was designed as to avoid as much as possible the actuators saturation under different performance. The control input $u(t)$ in this particular case represents the angular movements of the robot, so in this case it was taken as a scalar attenuation of 1/170 in all control channels. Finally, in order to minimize the error to step reference, and attenuate the input disturbances, the error weighting matrix $W_e(j\omega)$ must include an integrator. Here, instead of using this option, a first order lag filter was used in the three channels of the error vector with a pole very close to the imaginary axis. As a rule, this last weighting matrix is the main function for tuning the controller. Thus, after several simulations, it was obtained the following:

$$W_e(s) = \frac{10(s/5 + 1)}{(s/0.0021 + 1)}I_3$$ (13)

$$W_f(s) = \frac{10(s/5 + 1)}{(s/0.0021 + 1)}I_3$$ (12)

After application of the MATLAB Robust Toolbox, a high order controller $K_c(j\omega)$ was computed which was reduced, by truncation method to a low order controller (10th order), without much deterioration of the performance and stability conditions of the controlled system. The controller shows a high gain at low frequencies (20dBs) with a cross frequency around 0.08 rad/sec, and with a roll-off of -20 dBs/dec at intermediate frequencies.

The final step of the Hoo design is to check if the controlled system satisfies the nominal and robust conditions:

For robust stability:

$$\| -W_oS_oK_cN \| < 1$$ (13)

For good tracking, disturbance rejection as well as high noise attenuation:

$$\| W_oS_oW_nS_nW_pS_pK_c \| < 1$$ (14)

which is the condition for good nominal performance.

Finally, for robust performance the controlled system must satisfy the following condition:

$$\| \Gamma_{wz} \| < 1$$ (15)

The conditions for robust stability and nominal performance, were satisfied by the design as shown in Fig.6 below.
Robust performance analysis was carried out by means of the structural singular value of the close loop system. When this value is less than one then conditions of Eq. (15) is fulfilled. For this design a value of 0.7 was obtained, meaning that the controlled system stands up to 126% non-structured modeling variations. The time response of the controlled system to step reference inputs of 10, 15, and 25 degrees applied to the three joints of the robot respectively are shown in Fig. 7 for values of time delays of 0.8, 1.2, and 1.6 s.

Before the H∞ design was applied, a PI controller design was carried out on each of the loops of the NCS. It was found that for such controllers the system became unstable for a time delay in the communication channels as small as 0.34 s 7 times smaller than the value obtained by the H∞ design which was 2.1 s. Acceptable time responses were obtained for up to 1.2 s, as can be seen from Fig. 7. Therefore, the objectives of the design, of extending stability and performance of the system, for ample values of the time delay in the communication channels were satisfied.

4. Conclusions

Due to the new technology on wireless communication that has reduced cost, and size of the transmitters and receptors, as well as integrated them to sensors and actuators it is now possible to consider Network Control Systems. As is well know one of the main problems of such structure is the introduction of uncertain delays in the communication channels. While there exists delay stable independent systems, for those that behavior dependence on it, is important to extend the performance of the system as much as possible to ample values of the time delays. Norm H∞ optimization method is an effective way to deal with model uncertainty. Here this method has been applied to design a close loop wireless system and has shown significant improvement with respect to other design techniques. As a side benefit, the controlled system also showed great noise attenuation and good disturbance rejection.

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