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Applications of partial derivatives in economics: Consumer utility function (satisfaction with consumption)

Aplicaciones de las derivadas parciales en las ciencias económicas: Función de utilidad de consumo (satisfacción al consumo)

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Abstract:

The article "Applications of Partial Derivatives in Economics: Consumer Utility Function (Consumer Satisfaction)" explores the use of advanced mathematical tools to analyze and solve problems related to the utility function, a key concept in economics. Its main objective is to provide a practical guide to problems and solutions using partial derivatives to model and optimize consumption satisfaction as a function of multiple economic variables. Through a mixed approach, with qualitative predominance and descriptive, inductive and hypothetico-deductive methods, the accuracy of the analysis and applicability in real contexts is guaranteed. The study is based on reliable sources and the authors' teaching experience, presenting examples that illustrate utility maximization and the impact of budget constraints. This work fosters the development of analytical skills and critical thinking, in addition to promoting innovation in solving complex problems, contributing to meaningful learning in the economic sciences. The results demonstrate the importance of partial derivatives as a practical tool for understanding and optimizing consumption decisions, providing students and teachers with valuable resources for applying mathematical concepts in real economic situations.

Keywords:

Partial Derivatives, Utility Function, Consumer Satisfaction, Economic Optimization, Problem Solving

Resumen:

El artículo "Aplicaciones de las derivadas parciales en las ciencias económicas: Función de utilidad de consumo (satisfacción al consumo)" explora el uso de herramientas matemáticas avanzadas para analizar y resolver problemas relacionados con la función de utilidad, un concepto clave en economía. Su objetivo principal es ofrecer una guía práctica de problemas y soluciones que utilicen derivadas parciales para modelar y optimizar la satisfacción del consumo en función de múltiples variables económicas. A través de un enfoque mixto, con predominancia cualitativa y métodos descriptivos, inductivos e hipotético-deductivos, se garantiza la precisión de los análisis y la aplicabilidad en contextos reales. El estudio se basa en fuentes confiables y la experiencia docente de los autores, presentando ejemplos que ilustran la maximización de la utilidad y el impacto de restricciones presupuestarias. Este trabajo fomenta el desarrollo de habilidades analíticas y el pensamiento crítico, además de promover la innovación en la resolución de problemas complejos, contribuyendo al aprendizaje significativo en las ciencias económicas. Los resultados demuestran la importancia de las derivadas parciales como herramienta práctica para comprender y optimizar las decisiones de consumo, proporcionando a estudiantes y docentes recursos valiosos para aplicar conceptos matemáticos en situaciones económicas reales.

***Palabras Clave**:*

Derivadas parciales, Función de utilidad, Satisfacción al consumo, Optimización económica, Resolución de problemas

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Introduction

This paper is part of the doctoral thesis in progress entitled "Applications of partial derivatives in economics: a proposal based on problem solving". The purpose of this paper is to provide a practical guide to problems and solutions using partial derivatives to model and optimize consumption satisfaction as a function of multiple economic variables. Through a problem-solving approach, it seeks to provide clear and accessible tools that allow practitioners to address complex challenges effectively.

Currently, mathematics has diverse applications and is linked to other disciplines such as economics, medicine, engineering and physics. This makes mathematics a fundamental science for the development of models that address situations of the social context or any other kind [1].

One of the most outstanding applications of partial derivatives in the economic sciences is the analysis of the consumption utility function, a fundamental tool for understanding how consumers maximize their satisfaction from the goods and services available [2] [3]. This concept, widely used in microeconomic theory, allows us to evaluate the preferences and behaviors of individuals as a function of budgetary restrictions, variations in prices and changes in the availability of resources. Using partial derivatives, it is possible to calculate rates of change, analyze marginal relationships and optimize decisions, essential aspects for designing economic and business strategies.

This study addresses the existing problem in the careers of Economic and Administrative Sciences at the National Autonomous University of Nicaragua, Managua (UNAN Managua), Estelí Regional University Center (CUR-Estelí). The main objective is to establish the relationship between Mathematics and the careers of Economics, Business Administration, Marketing, Banking and Finance, and Public Accounting and Finance. In addition, it seeks to provide precise and concise material for the study of specific topics, such as marginal productivity, maxima and minimum of a function in several variables, and special attention is paid to marginal cost [4] [5].

The methodological approach developed in this research has as its main focus simplicity and clarity in problem solving. Through practical examples and detailed solutions, it is intended that the reader not only understands the mathematical concepts involved, but also acquires the ability to apply them in real contexts. This article, in particular, focuses on the treatment of the consumption utility function, addressing its definition, fundamental properties and practical applications through partial derivatives.

More and more economists and administrators consider that the use of mathematics, as a symbolic language and method of scientific reasoning, constitutes an invaluable aid in the tasks and objectives of these sciences. Its presence is fundamental both in the description of complex economic relationships and in the formulation of propositions about behavioral relationships [6].

The university is continuously dedicated to the integral development of students, integrating processes that go from the classroom to work in the communities, where they learn through practice. This benefits both parties in the educational process [7] [8].

The relevance of partial derivatives in economics transcends mathematical theory, since their effective application can directly influence strategic planning, market analysis and economic policy formulation [9] [10]. In a dynamic and competitive environment, having tools to predict and optimize results is a significant advantage. This article not only delves into the technical use of partial derivatives but also seeks to establish a tangible connection between mathematical theory and the practical needs of economics and management.

Development

From the point of view of the research approach, this study is considered a mixed study, since it is dedicated to describing and analyzing the problem in the economic and administrative areas. Throughout the process of interpretation, the research problem is reflected upon and deepened, employing both logical induction and an approach that moves from the specific to the general. Consequently, this study integrates both quantitative and qualitative elements [11] [12].

The collection of information aims to provide explanations on key topics for the advancement of knowledge. In this case, various sources, such as journals, books, websites, and PDF documents, were used to obtain data relevant to the content. In this study, a descriptive analysis of the information collected was carried out, starting with coding the data to facilitate its understanding. In addition, summaries, images, annotations and a quantitative analysis of cost functions and production problems were elaborated [1] [6]

Partial derivatives are fundamental tools in the economic sciences, especially in the analysis of the consumption utility function. This function represents the preferences of a consumer in relation to different goods and services, and its partial derivative makes it possible to evaluate the impact of changes in the quantity consumed of a good on total utility, keeping the consumption of another goods constant. By analyzing partial derivatives, economists can determine the marginal rate of substitution, which indicates how many units of one good the consumer is willing to give up obtaining an additional unit of another good, thus reflecting his preferences and consumption behaviors.

Moreover, partial derivatives are essential for utility optimization in the context of budget constraints [13]. Economists use these derivatives to identify optimal points of consumption, where the consumer's level of satisfaction is maximum given his income and the prices of goods. By setting up models that incorporate the partial derivatives, one can simulate economic scenarios and forecast how variations in prices or income affect consumption decisions. This is crucial for the formulation of economic policies and marketing strategies, as it allows decision-makers to better understand market dynamics and consumer behavior.

Even though tastes and satisfaction are familiar ideas, it is difficult to express them in concrete terms. Suppose you have just eaten an apple and a piece of candy, could you tell someone how much satisfaction you received from each? You might be able to tell which one you liked better, but could you express that in specific, numerical terms? [14]

The early neoclassicals reasoned as if utility or satisfaction derived from the consumption of goods was a quantifiable and aggregable phenomenon, as if the magnitude of utility was itself a relevant fact. It was thought to be a numerical measure of the happiness of the individual. Given this idea, it was natural to imagine that consumers made their decisions with a view to maximizing their utility, i.e., to be as happy as possible.

The consumption utility function, denoted in mathematics as $u(x, y)$ quantifies the level of satisfaction or utility that a consumer has when acquiring x units of a product. Often one is interested in all possible combinations of purchases that produce the same level of satisfaction c_0. In our terminology if we have the function $z = u(x, y)$ whose graphical representation is a surface in R^3, we are only interested in the trace with the plane $z = c_0$. This contour line given by the equation $c_0 = u(x, y)$ is called the indifference curve.

Application Problems

Problem 1: The monthly profit in Nuevo soles of a company that markets a single product is given by: $U(x, y) = \frac{1}{50}(x^2 + 2xy)$ where x represents the number of units sold in Lima and y the number of units sold in Arequipa. If the company currently sells 200 units in Lima and 300 units in Arequipa, estimate the approximate change in the company's profit if sales in Lima decrease by 1%, while sales in Arequipa increase by 2%.

Solution

Let the utility function be

$$U(x, y) = \frac{1}{50}(x^2 + 2xy) \tag{1}$$

$$\frac{1}{50} = 0.02 \tag{2}$$

$$x \rightarrow represents\ the\ number\ of\ units\ sold\ in\ Lima$$

$$200 - 1\% = 198 \tag{3}$$

$$y \rightarrow represents\ the\ number\ of\ units\ sold\ in\ Arequipa.$$

$$300 + 2\% = 306 \tag{4}$$

Substituting $x = 200, y = 300$ in the original function to calculate $U_1(x, y)\ y\ U_2(x, y)$ we have:

$$U_1(x, y) = 0.02[(200)^2 + 2(200)(300)] \tag{5}$$

$$U_1(x, y) = 0.02(40{,}000 + 120{,}000) \tag{6}$$

$$U_1(x, y) = 0.02(160{,}000) \tag{7}$$

$$U_1(x, y) = 3{,}200 \tag{8}$$

$$U_2(x, y) = 0.02[(198)^2 + 2(198)(306)] \tag{9}$$

$$U_2(x, y) = 0.02(39{,}204 + 121{,}176) \tag{10}$$

$$U_2(x, y) = 0.02(160{,}380) \tag{11}$$

$$U_2(x,y) = 3{,}207.6 \tag{12}$$

To calculate the approximate change in the company's profit it is sufficient to calculate

$$U_2(x,y) - U_1(x,y) = 3{,}207.6 - 3{,}200 \tag{13}$$

$$U_2(x,y) - U_1(x,y) = 7.6 \tag{14}$$

Answer: The approximate change is 7.6 utility.

Problem 2: Suppose the consumption utility function of two goods is given by $U(x;y) = x^2y$. A customer has bought 5 items of good x and 4 of good y. Represent geometrically other possibilities that the customer had to have the same level of satisfaction or utility in his purchase.

Solution

First, we calculate the customer's utility or satisfaction for this purchase. It is given by:

$$U(x;y) = x^2y, with\ x = 5\ e\ y = 4 \tag{15}$$

$$U(x;y) = (5)^2(4) = (25)(4) = 100 \tag{16}$$

Then we pose the indifference curve with:

$$U(x;y) = 100 \tag{17}$$

$100 = x^2y$, this is a curve in R^2. To better visualize the graph we write the equation as a function

$$y = \frac{100}{x^2} \tag{18}$$

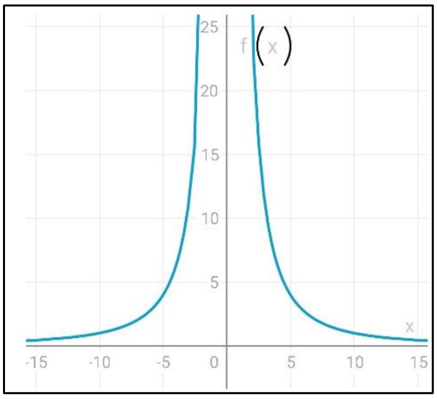

Figure 1. Indifference Curve

In Figure 1, each point of this curve gives the same level of satisfaction as buying 5 items of type x and 4 items of type y.

In the representation we only consider the positive part of the x's.

In economics it is common to determine different level curves for different quantities, in the case of cost functions, these curves are known as isocost lines.

Problem 3: A rural cooperative that produces inorganic and organic coffee. The cost of producing a kilo of inorganic coffee is 15 córdobas and organic coffee is 24 córdobas. The cooperative has fixed monthly costs of 4 000 cordobas.

a. Find the monthly production cost of both types of coffee.
b. If the cooperative places inorganic coffee on the market for 60 córdobas and organic coffee for 75 córdobas, obtain the utility function.

The cost of production of x kilos of inorganic and kilos of organic coffee is 15x and 24y respectively.

$$C(x,y) = fixed\ cost + variable\ cost \tag{19}$$

$$C(x,y) = 4\ 000 + (15x + 24y) \tag{20}$$

To find the utility function, we first find the total income function for the two types of coffee.

$$I(x,y) = sales\ of\ q_1 + sales\ of\ q_1 \tag{21}$$

$$I(x,y) = 60x + 75y \tag{22}$$

Finally, the utility is given by the difference between

$$g = U(x,y) = Revenue - cost \tag{23}$$

$$g = U(x,y) = 60x + 75y - (4\,000 + 15x + 24y) \tag{24}$$

$$g = U(x,y) = 60x + 75y - 4\,000 - 15x - 24y \tag{25}$$

$$g = U(x,y) = 45x + 51y - 4\,000 \tag{26}$$

Comment: The variables x, y are the independent variables while the utility function g is the dependent variable. As in the case of functions of one variable, the domain of the function must be specified so that it is valid in the field of real numbers. When dealing with functions of application to economics, the domain of the function must also make "economic sense".

<u>Problem 4:</u> The utility function $U = f(x,y)$ measures the satisfaction (utility) that a person finds by consuming two products x and y. Suppose that $U = 5x^2 - xy + 3y^2$

a. Calculate the marginal utility with respect to product x ($U_x(x,y)$)
b. Determine the marginal utility with respect to product y ($U_x(x,y)$)
c. When $x = 2\; e\; y = 3$, should a person consume one more unit of x or y to have more utility?

Solution

Let $U = 5x^2 - xy + 3y^2$, calculating the marginal utility with respect to x we have:

$$U_x(x,y) = 10x - y \tag{27}$$

Let $U = 5x^2 - xy + 3y^2$, calculating the marginal utility with respect to y we have:

$$U_y(x,y) = -x + 6y \tag{28}$$

Substituting $U_x(x,y)$ *and* $U_y(x,y)$ *in* $x = 2$ *and* $y = 3$ we obtain that:

$$U_x(2{,}3) = 10x - y = 10(2) - 3 = 17 \tag{29}$$

$$U_y(x,y) = -x + 6y = -2 + 6(3) = 16 \tag{30}$$

Answer: A person must consume one more unit of x to have more utility.

<u>Problem 5:</u> The utility function of a consumer with respect to two goods A and B is $U(x,y) = \ln(1 + xy)$, where x and y are the quantities, he consumes of the two goods respectively. Assume that the current consumption is $(x,y) = (10{,}10)$. It is requested:

a) Obtain the marginal utility with respect to good A. Interpret its sign.
b) Justify mathematically this statement: "For each unit that increases the consumption of A, the marginal utility of A decreases, i.e., the additional satisfaction of the consumer by increasing the consumption of A is less and less".
c) Justify mathematically this statement: "For each unit that increases the consumption of B, the marginal utility of A increases".

Solution

Calculate the marginal utility of good A considering the original function

$$U(x,y) = ln\,(1 + xy) \tag{31}$$

$$U_x(x,y) = \frac{y}{1 + xy} \tag{32}$$

Evaluating with $(x,y) = (10{,}10)$ in marginal utility we have:

$$U_x(x,y) = \frac{10}{1 + (10)(10)} \tag{33}$$

$$U_x(x,y) = \frac{10}{1 + 100} = \frac{10}{101} \tag{34}$$

$$U_x(x,y) \approx 0{,}099 \tag{35}$$

$$(x_0 = 10, y_0 = 10) \rightarrow (x_1 = 11, y_1 = 10) \tag{36}$$

$$\Rightarrow U_x(x,y) \uparrow 0{,}099 \text{ approximately} \tag{37}$$

To answer the situation of item b, the second partial derivative with respect to good A must be applied.

$$U_{xx}(x,y) = \frac{(y)'(1 + xy) - (y)(1 + xy)'}{(1 + xy)^2} \tag{38}$$

$$U_{xx}(x,y) = \frac{(0)(1+xy)-(y)(y)}{(1+xy)^2} \quad (39)$$

$$U_{xx}(x,y) = -\frac{y^2}{(1+xy)^2} < 0 \rightarrow x \uparrow (y\ constante) \quad (40)$$

$$\Rightarrow U_x(x,y) \downarrow \quad (41)$$

Note: This statement is valid for any point in the domain of the function.

To give a mathematical answer to this subsection, the mixed partial derivative must be calculated, i.e. $U_{xy}(x,y)$.

$$U_{xy}(x,y) = \frac{(y)'(1+xy)-(y)(1+xy)'}{(1+xy)^2} \quad (42)$$

$$U_{xy}(x,y) = \frac{(1)(1+xy)-(y)(x)}{(1+xy)^2} \quad (43)$$

$$U_{xy}(x,y) = \frac{1+xy-xy}{(1+xy)^2} = \frac{1}{(1+xy)^2} > 0 \rightarrow y \uparrow (x\ constant) \Rightarrow U_x(x,y) \uparrow \quad (44)$$

Note: This statement is valid for any point in the domain of the function.

<u>Problem 6</u>: A consumer purchases two goods in quantities x and y, so that her utility function is $U(x,y) = \sqrt{x} + \sqrt{y}$. It's current consumición level is $(x, y) = (9,4)$.

a. Calculate the current utility.
b. Write the equation of the current indifference curve and interpret it.
c. Calculate the Marginal Substitution Ratio RMS given by RMS = $-y'(x)$. Perform the calculation by implicitly deriving the equation. Interpret the value of RMS (9).

Solution

Substitute in the original function $(x, y) = (9,4)$

$$U(x,y) = \sqrt{x} + \sqrt{y} \quad (45)$$

$$U(9,4) = \sqrt{9} + \sqrt{4} = 3 + 2 = 5 \quad (46)$$

Since $\sqrt{x} + \sqrt{y} = 5$, then we can rewrite the equation as a function as follows:

$$\sqrt{y} = 5 - \sqrt{x} \rightarrow (\sqrt{y})^2 = (5-\sqrt{x})^2 \quad (47)$$

$$y = (5-\sqrt{x})^2, \text{ siempre que } 0 \leq x \leq 25 \quad (48)$$

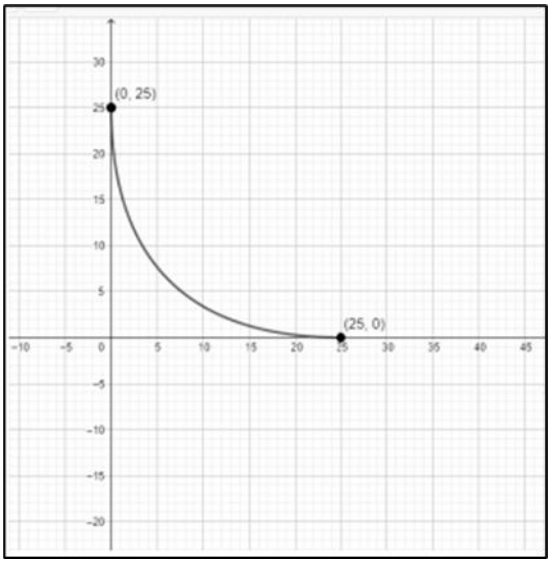

Figure 2. Indifference curve

In Figure 2, let $\mathbf{0 \leq x \leq 25}$ and $\mathbf{0 \leq y \leq 25}$ be the only points on this curve that give the same level of consumption satisfaction for $x = 9$ and $y = 4$.

Calculate the Marginal Substitution Ratio RMS, we derive with respect to x in the equation $\sqrt{x} + \sqrt{y} = 5$, which leaves us with:

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} * y' = 0 \quad (49)$$

$$y' = \frac{-\frac{1}{2\sqrt{x}}}{\frac{1}{2\sqrt{y}}} = -\frac{2\sqrt{y}}{2\sqrt{x}} = -\frac{\sqrt{y}}{\sqrt{x}} \quad (50)$$

If we know that $(x,y) = (9,4)$ and $\mathrm{RMS(x)} = -y'(9) = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3} \approx 0{,}66$

Therefore we can conclude that approximately 0, 66 units of good x are attributed to good y.

Conclusions

Partial derivatives, as fundamental tools in the economic sciences, allow modeling and analyzing complex problems related to consumer decision making. Through the detailed analysis of the consumption utility function, this paper demonstrates how advanced mathematics, specifically differential calculus, can optimize economic strategies and facilitate the understanding of market dynamics.

The results obtained highlight the relevance of partial derivatives for calculating marginal rates of substitution, identifying optimal consumption points under budget constraints, and assessing the impact of price and income variations on consumer decisions. These applications are not only theoretical, but have practical implications for public policy formulation, marketing strategies and business planning.

Furthermore, the methodological approach used in this research, based on problem solving and practical examples, allows for an accessible and applicable understanding of the mathematical concepts involved. This contributes significantly to meaningful learning, fostering analytical and critical skills in both students and professionals in the economic area.

In conclusion, this study reaffirms the importance of integrating advanced mathematical tools in the training and practice of economic sciences. The use of partial derivatives, as illustrated in this article, not only strengthens economic analysis, but also promotes innovation and optimization in solving real problems. This interdisciplinary perspective opens up new opportunities for the development of applied methodologies that benefit both the academic and professional spheres.

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