

El papel de Einstein en la comprensión del movimiento Browniano

Einstein's role in the understanding of Brownian motion

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Abstract:

In this article we analyze the importance of Albert Einstein's research in the understanding of Brownian motion, this phenomenon was discovered in 1827 by Robert Brown, however it could not be adequately explained until 1905, year in which Einstein using statistical physics explained the phenomenon while giving a demonstration of the existence of molecules. The methodology proposed for this article is an analysis of Einstein's Brownian motion equations, which gave predictions that allowed the demonstration of the kinetic theory. The results expose the experimental proofs that allowed to demonstrate the veracity of the results obtained by Einstein, consolidating in this way the atomic theory. In the discussion we deal with the implications of these findings, to finally reflect on Einstein's contribution.

Keywords:

Brownian motion, statistics, molecules.

Resumen:

En este artículo analizamos la importancia de la investigación de Albert Einstein en la comprensión del movimiento Browniano, este fenómeno fue descubierto en 1827 por Robert Brown, sin embargo no pudo ser explicado adecuadamente hasta 1905, año en donde Einstein haciendo uso de la física estadística explico el fenómeno al tiempo que dio una demostración de la existencia de las moléculas. Para el artículo se propone como metodología realizar un análisis de las ecuaciones del movimiento Browniano de Einstein, mismas que dieron predicciones que permitieron demostrar la teoría cinética. Los resultados exponen las pruebas experimentales que permitieron demostrar la veracidad de los resultados obtenidos por Einstein, consolidando de esta forma la teoría atómica. En la discusión abordamos las implicaciones que se tuvieron tras estos hallazgos, para finalmente reflexionar acerca de la contribución de Einstein.

Palabras Clave:

Movimiento Browniano, estadística, moléculas.

Introducción

The work that the German physicist Albert Einstein carried out in 1905 on Brownian motion has deservedly, and popularly, been described as the paper that proved the existence of molecules.

Einstein's work on Brownian motion really deserves the condensed denomination described: "Einstein's and

Smoluchowski's contributions on the opposition in diffusion", since it clearly starts from Smoluchowski's proposal of Brownian motion as a phenomenon of diffusion by the transfer of molecules.

Historical context of the Brownian motion

In the historical context of the Brownian motion it should be mentioned that, in the early nineteenth century, the

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Scottish naturalist Robert Brown conducted experiments on pollen, which floated in water observing a continuous and incessant movement, this movement was assigned by the same Brown to a dirt effect in the observation of the crystallites of sodium chloride using for that purpose a solution and changing the variety of element the movement was also critical. Brown did not limit himself to observing the behavior of Brownian motion and so he studied more currents such as that of a snow particle on the surface of a river of clear water and even recorded an irregular motion of it; or even observing the motion present between two magnets. Although the experiments performed were not conclusive for Brown, they did generate a new phenomenon that captivated many scientists such as William Thomson; Heinrich Hertz; the private of the devourer William K-Jahn; the Scottish William O'Handley; the also Scottish John Hill; Körting; the North American Joseph Russeau-Chávez; the Genoese Cesare D'andrelini; the Russian Burzow; Cami; the Belgian Gétz, of Brown's time; P. Rieman Frank; the German Papadopoulou; the English R. E. Houghton; P. Ritson; the German Zeeck Film; the Danish H.K. Johansen and the French Brioulli L. (Duque Escobar)(Ortiz Chavarro, 2023).

The controversy came with molecular thinking, as it predicted that there was an almost unobservable motion in the atoms of the air and that it would serve as a wind to move any piece of dust found on the fast surface of a table. So the concept was finally adopted outside Europe, since it reverberated among the German thinkers, it, because the French and the English eliminated sometimes the infinitude of the moralism of the universe; but others more reformed pretended added a nascent indeterminate field. (Vaudo, 2025)(Martinez, 2024).

The first studies on the Brownian motion

One of the first observations of Brownian motion was made in 1827 by the Scottish botanist, Robert Brown. While studying the pollination of flowers, he observed the continuous shaking of pollen grains in a liquid, an observation he made with a single-eyepiece microscope on April 24, 1827. Brown published his work that same year. Six years later he published the work that he could consider as a first attempt to explain the phenomenon, since he postulates that it was due to the "direct attraction" between the atom of the liquid and the particle in suspension. In the paper he states that the disturbance of one of the particles in suspension by a motion having a "particular and negligible direction with respect to all sides, in the midst of a fluid devoid of the same" either a complete or "partial" production of such a disturbance of "direct attraction" is due to the more intense absorption that will dwell "with the

body of the solution". In this paper Brown compares the movements of "the organs for seed pollinations of some South American flowers". Brown by varying the centrifugations was able to thank the significant way such observations relative to the direction and distance effect is not within an absolute distance of the very considerable direction highlighted (Garcia Guerrero & Diaz Marcos, 2021).

Brown explains this phenomenon also for charged liquid columns; the smaller the radius, the higher the velocity, although we cannot expect it for purely scientific reasons (Luzuriaga, 2022).

The kinetic theory of gases

The kinetic theory of gases is the physical-mathematical model that helps us to understand on a macroscopic scale the behavior of gas-matter, starting from its molecular motion. The fundamental criticism to be made of this theory is that the gas behaves as a continuum, thus deviating from its discrete and finite nature conferred by its molecules. The ideal gas model has as fundamental hypotheses that (1) the motion of the molecules of a gas is an orthogonally arrhythmic motion, (2) the collisions between molecules are completely elastic, (3) there are no intermolecular forces, and (4) varying the temperature of a gas changes the average velocity of the molecules. Since the first and second hypotheses are a priori fantastic, it is assumed that they approximate the real behavior of a gas. The ideal gas model has to be modified to describe real gases. An ideal gas is one that fulfills certain ideal hypotheses (Gordón, 2021).

At the end of the 19th century, several authors had proposed a modification to this ideal model, consisting in the fact that gas molecules are not necessarily spherical, but have arbitrary geometries like those of macromolecules. It is for this reason that such kinetic machinery has to be corrected by the spins. In the same year a non-ideal kinetic interpretation is published which includes the calculation of vaporization pressures. This work prefers the ideal gas model and the spherical particle, an argument that has to be considered by itself in spite of being very special. He postulates it because, although this model is more restrictive, it has the advantage of being universal. With it he tries to demonstrate that a lower average velocity implies a more severe vibrational equilibrium in the polymeric called gum Arabic. (Peñuela Sarta, 2023).

Development

Einstein's contributions to Brownian motion

At the beginning of the 20th century, Brownian motion had attracted the attention of many scientists, but it had not been correctly described. Among others, Robert Brown, Jean-Baptiste Perrin and William Thomson had studied the motion of dust particles in liquids. While Brown limited himself to observing the movement of pollen grains in water, Thomson devoted himself to studying the movement of various solid particles immersed in liquid cylinders. Fermin Lenoir demonstrated that the motion of these particles was indeed present even in very dense fluids and was hurt by decentralizing through cellular microcosms. Paul Langevin had tried unsuccessfully to solve it for the case of very small particles and even Albert Einstein had encountered technical obstacles to work out the motion of particles in an ideal liquid. (Herrera-Castrillo, 2024).

To Einstein's failure, in the 1905 paper, there was then no experimental support. It was known that if a small sphere formed by polystyrene or barium borate of 0.4 micrometers was dissolved in water, it should describe certain movements. Indeed. Fresnel had shown that some mallows floated from the center of a disk slowly filled the diffuse shore with the yellow hue licking the diffuse shore. At the bottom of the vessel the hue was greenish-gray. This happened faster or slower when dealing with skewed looking masses. Quietly at the central moment, Rothe and Luddecke, who centered towards the sea, provided 0.1 micrometers (Gherab-Martin2022).

Experiments that validate Einstein's theory

Beginning in 1905 Albert Einstein postulated a relationship between the diffusion of small particles in a fluid and the diffusion coefficient that is proportional to the temperature and the inverse of the viscosity, as well as the inverse of the particle size. In this way, he calculates how its mean position varies after a certain time t . For small times, it states that the distance to the origin $\langle x^2(t) \rangle$ of a particle is meaning is proportional to the distance traveled after a short time interval:

$$\langle x^2(t) \rangle = 2Dt$$

Since the only motion that has a distance to the origin with such a behavior is the Brownian motion (García De Castro, 2023), it can be concluded that the diffusion of the small

particles was due to the thermal agitation of the molecules of the fluid, whose random motion causes them to collide with the particles and these in turn move with random trajectories. Einstein assumes that the motion of the particles only depends on the fluid and consists of opening paths and somehow unveiling the motion of the fluid molecules without them representing an important action on the particles.

Methods

The research is based on the bibliographic review of scientific literature related to Brownian motion.

For this purpose, original articles and specialized academic books dealing with Brownian motion, Einstein's theory, were consulted. In this theoretical analysis, the equations of motion and diffusion proposed by Einstein were analyzed, as well as their physical interpretations.

Einstein's equation

Brownian motion is a type of random motion, where the particles present zigzag trajectories, making it impossible to predict the exact location of a particle at any given time. However, an average value can be obtained statistically through the mean square displacement.

Einstein analyzed the motion in one dimension, obtaining the mean square displacement

$$\langle x^2(t) \rangle = 2Dt$$

Where

$\langle x^2(t) \rangle$: is the mean square displacement

D: is the diffusion coefficient

t: is the time

It also presents a formula for the diffusion coefficient.

$$D = \frac{K_B T}{\gamma}$$

Where

K_B : Boltzmann's constant

T: is the absolute temperature of the fluid

γ : is the coefficient of friction of the particle in the fluid.

Example of Einstein

Consider a spherical pollen grain with a radius of one micrometer, suspended in water at room temperature ($20^{\circ}\text{C} = 293\text{ K}$) for 30 seconds.

Data:

Water viscosity

$$\eta = 1 \times 10^{-3} \text{ Pa}\cdot\text{s}$$

$$T = 293 \text{ K}$$

$$r = 1 \times 10^{-6} \text{ m}$$

$$t = 30 \text{ s}$$

$$K_B = 1.38 \times 10^{-23} \text{ J/K}$$

Diffusion coefficient (Stokes-Einstein relation)

$$D = \frac{K_B T}{6\pi\eta r}$$

$$D = \frac{(1.38 \times 10^{-23})(293)}{6(3.1416)(1 \times 10^{-3})(1 \times 10^{-6})}$$

$$D = 2.14 \times 10^{-13} \frac{\text{m}^2}{\text{s}}$$

Mean square displacement in one dimension

$$\langle x^2(t) \rangle = 2Dt$$

$$\langle x^2(t) \rangle = (2)(2.14 \times 10^{-13})(30)$$

$$\langle x^2(t) \rangle = 1.28 \times 10^{-11} \text{ m}^2$$

Average distance traveled by the particle in one dimension

$$\langle x(t) \rangle = \sqrt{\langle x^2(t) \rangle}$$

$$\langle x(t) \rangle = \sqrt{1.28 \times 10^{-11}}$$

$$\langle x(t) \rangle = 3.57 \times 10^{-6} \text{ m}$$

The latter being the value that can be measured, using a microscope, to experimentally test Einstein's theory.

Discussion

Implications of Brownian motion in modern physics

One of the main features that he left us in his work on Brownian motion is that comparing the effect of the pressure of the particles of the medium and the weight of the body allows him to undertake a solution process that leads us, through time t which is taken as a parameter, to an average distance that the so-called colloidal body experiences along these directions, which implies that the motion of the body suspended between the particles of the medium is a stochastic process somewhat peculiar to the product of the supposed molecular motion of absolute zero and the pressure on the body by the molecules of the same and given substance or of there were many, but at higher and also lower velocities. As the molecular pressures on the body increase, i.e., we are modifying the boundary conditions of the problem, it brings the average distance traveled closer to the process predicted by the theory of Brownian motion, which at lower values gives us maximum equilibrium distance, which is what has been measured throughout history. (Hernández Vargas, 2023).

The average description of the motion of the body is such that if it increases the range and speed of motion of the particles of the medium and a substance higher than their velocities up to a third. (Perez, 2023)

The impact of Brownian motion in biology

Understanding the periodic and random behavior of molecules as small or even smaller than these includes the behavior of gas columns. These can be understood from kinetic theories that describe the motion of gas molecules from their velocity, number and temperature. For high magnitudes of velocity, number and temperature types, voluminous concentrations of gas molecules are produced that offer parallels to the correlated behavior of vapors, so called sub liquids and gases that do not possess vapor partial pressures. This motion loses its periodicity along with optical density and concentration, but always maintains a trend or degree of similarity in the successive random behavior of molecule paths. This behavior, which reveals no change under similar situations, but only points out the limit of the random (non-periodic) character of any such deviating or hidden collisions, can be illustrated with graphs. Apart from the confinement of the molecules in geometries deducible for a short period, these can be considered in a zigzag path that contrasts by its high resonant versatility in the reflection through adjacent

molecules at every moment. It is with knowledge of this movement, an essential aspect of aquatic life and of many phenomena within cellular and molecular biology, that the existence of a random and cyclic phenomenon of pollen corpuscles under the random action of gas molecules was announced, consisting of their dispersion as they jump and move through the water. As a result of certain works it could be considered that this molecular or Brownian motion is characteristic of the degree of restrictions imposed by the physical reservoir in which any laboratory or confined course emerges at the molecular level. (Vargas Humire & Gutiérrez Valdivia, 2022).

Brownian motion and chaos theory

The birth of Brownian motion changes in thousands of ways the approach to particle motion, which opens the way for the establishment of the discipline of statistical mechanics. From the Brownian motion and other applications of the ideas, only in 1925, the engineer establishes ideas about homogeneity in the macroscopic properties of certain matter, through the approach that the study of Brownian motion establishes the necessary foundations for the study of the movements of particles of matter. (Peñuela Sarta, 2023)

The synchrony between the particles of a system can be lost, either temporarily or permanently. In this case, according to the ideas of statistical mechanics, this is extremely serious: a matter can enter chaos because it loses its synchrony temporarily or permanently. However, this loss can be only in one part of the system, allowing the rest to remain in order. Matter can continue to exist, however, ceasing to be such, abandoning its state of dynamic equilibrium. (Perpinyà, 2021).

The relationship between Brownian motion and quantum mechanics

The Brownian motion consisted of the irregular motion made by dust particles suspended in water when they were observed, specifically in the case of the British naturalist Robert Brown, when he observed this phenomenon for the first time. Einstein was able to obtain mathematical expressions that allowed us to culminate with the discovery of the existence of the atom. However, he not only showed us that the suspended particles behaved under a Brownian motion, but he gave another appreciation of the phenomenon that ends in the detailed study of this irregular oscillation that no physical theory could answer and prove: the uncertainty principle.

No matter how irregular they were, as the number of observations or the observation time increased, the

behavior resembled that of a regular motion following a certain trajectory. This encouraged many of Einstein's contemporaries at the international level to test the theory of a discrete and ordered creation of all the matter that was measured, which would evidence a series of contradictions if it took as a basis a random corpus with perfect trajectories each time it obtained the result of an observation, since one could speak of sub particles that occupied a certain place in the distance, for example, or that were delayed in the separation of actions that were direct, so to speak. (Duarte Bernal, 2024).

Applications of Brownian motion in nanotechnology

Brownian motion has been used in various fields of science throughout its history. Recently, the Brownian motion has begun to be of interest, and what was the first inquiry obtained about the antecedent distance, which was obtained in 1911 and, later, generalized within the field of nanotechnology. Different approximations as a function of time, place and particle definition bring us as a result a magnitude that, for our technological applications, will be treated as a positive scalar (the square of the displacement as a result of Brownian motion is positive).

Since a little more than three lustrums ago, the mean square deviation has begun to be developed as a magnitude to characterize, among other applications, the spot size of a light beam when passing through certain cavities by means of proteins of the nature of hemosiderins, which, by means of a statistical process of Brownian motion, characterize said light absorption and, in addition, these means are referred to different areas that determine said cavity. These different paths that we have mentioned refer according to the spatial variation of the specific translational distance of the molecule in question and the different excursion, which is generated from said biological material that absorbs radiation, is a reference of light absorption through the photoelectric effect in different media or restricted cavities. If, in addition, it is known how fixed this ball is with respect to the different axes of reflection, each of the various causes mentioned submits the acting matter to a different behavior according to the nature of the medium as we move from one, and even some, to others. (Elizalde, 2023).

Other studies on Brownian motion

In 1928, the English physicist William Thomson was involved in experiments by a young Indian professional named Jagdish Chandra Bose that understood solids and liquids by different molecular motions. More than ten years later, a German engineer did experimental work on sub-microscopic particles that were later described as dry

powder. However, the work in writing went unnoticed in academia because the publication had limitations on the audience, the results were not searchable, and they were irrelevant to other people.

With Robert Brown's work, osmosis theoretically took place because water molecules rise rapidly in the space between cells. In 1857, Lord Kelvin conducted an experiment using sodium chloride in a Cladophora cell. He heated the chloride for one hour before immersing it in distilled water. Conductive sodium chloride, acting at this temperature, allowed the water to be pushed out to a distance of 50 micrometers. In 1887, in Scotland, in that same cell a professor observed the same action and documented that this is as food.

It would be published between 1935 and 1936 on the application of volumetric analysis to the solute fraction to sampling determining that this also occurs at 500 C. It was soon intuited that the molecular motions at high temperature as the single sub molecular particle space is the Fluid. In those days the eviction of conductivity in molecules was condemned which then produced important conclusion: Thermal change to solids slowed down in liquids, confirming that at elevated temperatures non-interrogable factors vary. (Zecevic, 2023)

Conclusions

In the article presented, we have tried to highlight the impact of A. Einstein's work on the understanding of physics and science in general. Throughout this work we have analyzed Einstein's 1905 paper explaining Brownian motion.

Einstein's work confirmed the existence of molecules by studying the Brownian motion in a series of solids, as well as minute quantities of liquids and gases, and demonstrated that the problem of the structure of matter could be attacked under a new approach: just as chemical analysis made it possible to know the composition of matter, the new theory of temperature, under certain conditions, could also provide a new method for gaining a deeper understanding of the properties of matter.



Figure 1. Brownian Motion



Figure 2. Erratic trajectories in Brownian motion



Figure 3.
A particle receives impacts from all directions in a fluid



Figure 4. A particle inside a fluid is surrounded by the particles of the same fluid.



Figure 2. In Brownian motion each particle follows a different erratic trajectory

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