

Límites de la velocidad de una partícula

Limits of the velocity of a particle

Alfredo Olmo Hernández^a Reyna Romyna Olmo Hernández^b

Abstract:

In this article, a detailed and exhaustive analysis of the limits to the velocity that a moving particle can attain is carried out, considering in a thorough and profound manner both the classical theoretical frameworks and the important and significant relativistic and quantum implications related to these physical theories. These theories impose fundamental and essential constraints on particle motion, altering and transforming our understanding of physics at both micro and macroscopic levels in the universe.

Keywords:

Relativistic mass, relativistic contraction, Planck density, velocity, quantity of motion, energy.

Resumen:

En este artículo se lleva a cabo un análisis detallado y exhaustivo de los límites de la velocidad que puede alcanzar una partícula en movimiento, considerando de manera minuciosa y profunda tanto los marcos teóricos clásicos como las importantes y significativas implicaciones relativistas y cuánticas relacionadas con estas teorías físicas. Estas teorías imponen restricciones fundamentales y esenciales sobre el movimiento de las partículas, alterando y transformando nuestra comprensión de la física a niveles tanto micro como macroscópicos en el universo.

Palabras Clave:

Masa relativista, contracción relativista, densidad de Planck, velocidad, cantidad de movimiento, energía.

Introduction

In colloquial language, velocity is associated with the speed of a motion. The dictionary of the Royal Spanish Academy defines velocity as "speed, lightness or alacrity in movement". In physics, a more precise meaning is given to this notion. In Newtonian mechanics, we consider the motion of a particle, i.e. of a body that can be modeled as a point. One can speak of average velocity, which is defined as the quotient of the displacement and the elapsed time.

In this sense, a velocity can be high or low, without reference to an extreme value. The Boltzmann-Sinai theorem states that any macroscopic body in a confined

space of sufficiently large size exhibits a quasi-periodic motion, which can be assigned a "critical" velocity, which can be understood as the maximum velocity it can reach.

In the kinematic theory of a vehicle, the velocity that is estimated as maximum is no more than a "theoretical maximum velocity". In this case, the simplified model ignores air resistance, tire friction, vehicle inertia, etc. Therefore, in practice, vehicles cannot reach this speed, and the limit is much lower and depends on the forces acting on the vehicle. A train running on rails can reach much higher speeds, as the friction in this case is practically negligible. However, the speed limitation is not linked to the type of

^a Autor de correspondencia, Colegio de Bachilleres del Estado de Hidalgo | COBAEH CEMSAD Huitzila | Tizayuca-Hidalgo | México, <https://orcid.org/0000-0001-9778-6588>, Email: alfredooh16@gmail.com

^b Colegio de Bachilleres del Estado de Hidalgo | COBAEH CEMSAD Huitzila | Tizayuca-Hidalgo | México, <https://orcid.org/0000-0002-9733-073X>, Email: londonlun12@gmail.com

model, but to the energy that can be contributed to the system.

Theoretical framework

In classical physics velocity is defined as the change in position of a body divided by the time interval in which the change occurs. It can be an average or an instant. Conceptually it could also be a limit, i.e., the value that the velocity takes when the time interval tends to zero. However, the latter sense should not be confused with the critical velocity, i.e., the theoretical maximum velocity of a moving body. In practice, the critical velocity approaches the maximum value of the velocity that a body can reach, although it never supplants it. This value is theoretically equal to that of light. In practice it cannot because of difficulties of matter and energy, i.e., because of the physical support capable of supporting and maintaining such velocities. In subatomic particles, where the support ceases to intervene, the limit is different.

Special relativity indicates that as the velocity of an object increases, the time that elapses for the object dilates with respect to an external observer, and the length of its displacement contracts. These two effects go in opposite directions and balance each other, but the first one indicates that when moving at the speed of light time for the object itself ceases to exist. The speed of light is also invariant for all observers and, therefore, it can be said that light does not travel at maximum speed, it simply does not travel. The Planck scale, in which values of microscopic magnitudes arise, also introduces a speed limit, which in this case is purely conceptual. It is the Planck density, which results from the division of the Planck mass by its volume. Solving the problem of any object capable of moving faster than light is also a challenge. The definition of causality, in which an effect cannot precede its cause, is derived from relativity, from the analysis of space-time structure and goes beyond relativity. (Paredes, 2024)

Concept of velocity and maximum velocity in classical physics

In physics, we speak of velocity when we take into account the displacement of a system during a time interval, associated with the notions of displacement and time. The notion of critical velocity comes from the study of the acceleration of a body in a medium whose resistance increases with velocity, such as air or water. In general, physicists know the velocity in space as the critical or theoretical maximum velocity, and the average velocity and instantaneous velocity are common concepts applied to many phenomena. The velocity in nature has maximum

values, and one of these values is that of light in vacuum, which is denoted as c and is 299 792 458 m/s. (Merma, 2022)

Theoretically, it can be said that, unlike macro-bodies, which are the structures we see in everyday life and which are composed of an immense number of particles, particles are systems whose velocities of motion have definite limits. In the case of macroscopic particles, following common sense, when the velocity of a body increases it is expected that, reaching a certain value, the acceleration stops being positive and begins to be negative, due to the effect of the media involved in the movement, such as air resistance and friction. An example of this is the terminal velocity model and its applications to falling bodies. However, mathematical models of particle acceleration in the direction of motion, and explaining the shape of flying objects in the air, show that, in reality, there should be no maximum velocity of everyday life, and that, therefore, it should be possible to exceed the speed of sound, in air, or of light, in space.

Special theory of relativity and its impact on velocity

The special theory of relativity, formulated by Albert Einstein in 1905, is one of the fundamental bases of modern physics. Its content has been confirmed by experiments, and its logic is derived from the invariance of the speed of light. While recognizing the importance of this invariance, the theory itself encompasses much more than a simple invariance; among other things, it allows us to understand why the speed of objects cannot exceed the speed of light. The interpretations of relativity in relation to velocity are diverse, and are presented here as an aspect of the causality principle outlined in the introduction. (Herrera-Castrillo, 2024)

The special theory of relativity states that the velocity of a particle is limited by the speed of light in a vacuum, and that as its energy increases, its relativistic effects become more pronounced, affecting its proper time, measured length and apparent mass. (Merma, 2022)

Relativistic mass in modern physics

Relativistic mass refers to the increase in the apparent mass of an object, moving at speeds approaching the speed of light. (Gualteros Rojas, 2023)

Relativistic mass is defined as

$$m = m_0 \gamma$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Where

m = Is the relativistic mass (or apparent mass) of the particle when moving at velocity v .

m_0 = Is the rest mass (or invariant mass) of the particle, which is the mass measured by an observer who is at rest with respect to the particle.

γ = Lorentz contraction factor.

v = Is the velocity of the particle relative to the observer.

c = Velocity of light in vacuum.

Relativistic contraction or reduction of length in motion.

The theory of special relativity states that the length of an object moving at a significant velocity relative to an observer is measured as shorter in the direction of motion than its actual length measured at rest. (Sobel, 2023)

Relativistic contraction is defined as

$$L = \frac{L_o}{\gamma}$$

Where

L = Is the length measured by an observer moving with a certain velocity v relative to the object..

L_o = Is the length measured by an observer who is at rest with respect to the object.

Planck scale.

The Planck scale is a conceptual limit presented in the physical model most commonly used to describe the physical world. At this scale, the appearances of the effects of quantum mechanics and relativity, especially the theory of black holes, suggest that the manipulation and measurement of certain quantities should be carried out with great caution. These ideas do not correspond directly to causal constraints. The analysis of causal limits on the Boltzmann-related particle is, in fact, based on causal constraints arising from special relativity. Causal bounds are of particular importance for the standard model of contemporary particle physics. From it a causal limit is derived on the motion of macroscopic bodies and on the motion of particles of quantum field theory that are considered particles of the observed universe, such as electrons, protons or neutrons. (Rincon Paez, 2025).

Planck density

The Planck density is a fundamental unit of density within the Planck system of units. It represents the maximum value of density that has meaning in physics, combining the fundamental constants of nature. (Akoudad Ekajouan, 2021)

This unit is obtained by combining the universal constants of gravity, the speed of light and the reduced Planck's constant, thus defining a scale on which the effects of quantum mechanics and general relativity become equally important and dominant.

The Planck density is defined as

$$\rho_P = \frac{c^5}{\hbar G^2}$$

Where

ρ_P = Planck density.

\hbar = Planck's reduced constant.

G = Universal Gravitation Constant.

Causal limit and its origin

Even without formalizing the structure of a quantum model of gravity, the physical nature of theories beyond the standard model suggests the existence of a velocity limit on particle motion. In the studies of Garcia et al. this limit is associated with causality, considered as a more general principle than the relativistic invariance of the speed of light.

The concept of causal limit comes from relativistic causality, which guarantees that spatially separated events cannot influence each other. Relativistic causality is intimately associated with the structure of space-time and ensures that causality is respected between two moving classical particles subject to interactions. Therefore, a velocity limit stemming from relativistic causality could be derived from an analysis of the structure of space-time. (Barrios)

Models and approaches for massive particles

In classical physics, the motion of macroscopic bodies is modeled as the displacement of a particle. The description of the motion is conditioned by the existence of a medium in which the waves propagate and by the existence of forces that can act on the body. In general, the velocity of these

bodies is bounded by a critical velocity, such as the speed of sound in a given medium, which they cannot exceed. In special relativity, particles are defined as objects that constitute an inertial reference frame and possess inertial mass. Photons are the only particles that do not have inertial mass, so they always move at the speed of light in a vacuum. However, other particles may be involved in their motion that, although they have non-zero inert mass, move at speeds very close to the speed of light. (Juárez Rojas) Beyond the standard particle model, there are theories that attempt to describe phenomena that cannot be replicated in laboratories, such as gravity on a quantum scale, but a convincing conclusion has not yet been reached. These theories also attempt to unify gravity and quantum mechanics. If consistent and coherent models of particles at velocities greater than the speed of light can be constructed, they could make sense of the interpretation of experimental results, although there is currently no theory that allows a description in these terms. (Paredes, 2024)

Particles in classical physics

A particle, in classical physics, is an object that can be modeled as a material point: that is, it possesses mass and occupies a space-displacement that can be considered negligible compared to the rest of the system. Therefore, it is not necessary to consider its internal structure and microscopic aspects, but its degree of freedom and its macroscopic constitution, which limits the speed at which it can move, must be taken into account. Generally speaking, the motion of macroscopic bodies will be described by the use of forces, and the friction of the media is usually used as the main speed limiter. For the motion of particles possessing an inertial mass, the velocity limit is considered to be given by the model of a body subjected to a constant force which, in the case of gravitational acceleration, is approximately equal to c . (Janssen, 2022)

In general, particles are bodies of such small dimensions, relative to the values of space and time used, that the distance they travel during a time interval is not perceived and their motion is considered to be instantaneous. However, there are some limitations to the motion of particles, since they cannot be perfectly motionless and, therefore, cannot reach infinite velocities or velocities greater than the speed of light.

Particles in special relativity

Special relativity describes the behavior of objects at velocities close to the speed of light. Particles in this theory are entities that can move at or below the speed of light, and their nature reflects these characteristics. Such a

classification is applicable to macroscopic objects, in which inert mass is the only one considered. Photons are the only ones to move at light speeds; although they are not usually assigned a mass, this can be defined as zero. The speed of other objects approaches the speed of light, but never reaches it. (Paredes, 2024)

A particle can be considered as a body that follows a defined trajectory in space through time. For a macroscopic object, its trajectory can be described, as in classical physics, in terms of the three spatial coordinates. However, the Planck density and the small scale of quantum mechanics suggest that, at least theoretically, it could not be constructed by an infinite number of particles of zero inertial mass. (Canals & i Valls, 2023)

Particles in theories beyond the standard model

In several theories beyond the standard model of particle physics, limits to the velocity of particles are discussed. In general, particles modeled as macroscopic bodies in classical mechanics have a velocity limit in the absence of gravitational accelerations. Particles modeled as points in special relativity, on the other hand, have an invariance of the speed of light as well as an inertial mass of zero or tending to zero in the case of photons. In these contexts, velocities close to the speed of light c do not violate any physical reason. In general relativity, which encompasses special relativity as a particular case, in addition to relativistic causality it is possible to deal with the effects of time dilation and space contraction. In theories that supersede special relativity, such as the lightning theory of quantum gravity, relativistic causality may not be present (Reguera).

In the standard model of particle physics, on the other hand, particles are modeled as point objects. In these theories, particles traveling at superluminal velocities are generally not expected to exist. However, in the standard model the quantum nature of matter is highlighted by the properties of the wave function. Although for traditional quantum mechanical particles it has been established that they cannot be detected at time intervals that tend to zero, there are more advanced models that use these limits as a property to be considered in experiments.

Methods to approximate velocity limits

Experimental methods exist that allow particles to be accelerated and their interactions to be detected, giving clues as to the speed at which they are moving. These methods, although not suitable for determining limits, can provide indirect evidence suggesting the existence of limits

for certain particles, such as electrons. These limits, whether by means or by forces, are not absolute, but at best represent observable limits. In general, the observation of a phenomenon may be determined by limits imposed by the means that allow observation (in relativistic phenomena, by the speed of light) or by the physical model that describes that phenomenon and the precision with which that observation can be made (e.g., the probabilistic model in quantum mechanics). In this framework, there are two complementary approaches that allow to obtain limits for the velocity of particles that are faster than light in some medium: one deterministic and the other probabilistic. To the first belongs the Cerenkov principle, which guarantees that when a particle of electric charge moves faster than light in the medium, the photon emitted in the Cerenkov interaction can be detected. To the second belongs the Tamm-Frank principle, which guarantees that, although a system cannot emit radiation because one of its parts is moving faster than light, the interaction of that system with others can be described by radiation. (Berjón and Okon2024)(Salvat, 2023)

Acceleration and detection experiments

The estimation of particle velocity limits is based on acceleration and detection experiments. For these tests, a precision criterion of 0.001c or better is preferred, and errors are determined using the Monte Carlo method.

(Paredes, 2024)

To establish velocity limits, experiments that neutralize the influence of conditions at the time of detection are required. In this sense, acceleration and later more sophisticated acceleration models are the most appropriate methods. The essence lies in studying the acceleration velocity of particles. In these experiments, although detections are performed, the main feature is that the acceleration is the process of interest. Therefore, they are developed as a function of acceleration rather than detection.

Theoretical interpretations of observable limits

Different acceleration and particle detection methods allow estimating speed limits by determining their corresponding travel times. In general, an accuracy comparable to a quarter of a travel time is required for the deduced velocity limit to be meaningful. However, such accuracy criteria are neither strict nor always met, and under certain conditions there are interpretative differences depending on the method employed.

On the one hand, in the deterministic domain, velocities are not mere approximations to a value, but possess a definite and deterministic value that, under ideal conditions, can be

inferred from the measured times. On the other hand, in the context of quantum mechanics, the probabilistic nature of the phenomenon allows that two particles can be quantum correlated during a trip and that the causal relationship is expressed in terms of detection probabilities. (Chaves & Garcia)

Determination of the limits of the velocity in a particle

Let a particle with a quantity of motion be a particle. It will be shown that the particle has a limit to its velocity, where the limit depends on its physical properties. It will also be shown that for every inertial observer there is a limit value that can measure the amount of motion and energy.

The following postulate will be used as a starting point:

The Planck density is the highest possible density.

The particle is considered to be perfectly spherical, with homogeneous density.

We proceed to obtain the density of the particle while it is at rest.

Being a sphere with homogeneous density we have that the density of the particle is:

$$\rho_o = \frac{m_o}{V_o}$$

$$V_o = \frac{4}{3} \pi r_o^3$$

Where

ρ_o = Density of the particle at rest.

m_o = Mass of the particle at rest.

V_o = Volume of the particle at rest.

r_o = Radius of the particle at rest.

$$\rho_o = \frac{3m_o}{4\pi r_o^3}$$

Now, the particle by presenting a quantity of motion, is in motion. For the relativistic mass and relativistic length contraction we have:

$$m = m_o \gamma$$

$$p = p_o \gamma$$

$$r = \frac{r_o}{\gamma}$$

Substituting to obtain the relativistic density.

$$\rho = \frac{3m}{4\pi r^3}$$

$$\rho = \rho_o = \frac{3m_o \gamma^3}{4\pi r_o^3}$$

$$\rho = \frac{3m_o \gamma^4}{4\pi r_o^3}$$

$$\rho = \rho_o \gamma^4$$

Since the Planck density is the highest possible density, we have that for any velocity it is satisfied that

$$\rho = \rho_o \gamma^4 \leq \rho_P$$

It follows that

$$\rho_o \gamma^4 \leq \rho_P$$

$$\gamma^4 \leq \frac{\rho_P}{\rho_o}$$

$$\frac{1}{(\sqrt{1 - \frac{v^2}{c^2}})^4} \leq \frac{\rho_P}{\rho_o}$$

$$\frac{1}{(1 - \frac{v^2}{c^2})^2} \leq \frac{\rho_P}{\rho_o}$$

$$\frac{1}{1 - \frac{v^2}{c^2}} \leq \sqrt{\frac{\rho_P}{\rho_o}}$$

By properties of the inequalities.

$$1 - \frac{v^2}{c^2} \geq \sqrt{\frac{\rho_o}{\rho_P}}$$

$$1 - \sqrt{\frac{\rho_o}{\rho_P}} \geq \frac{v^2}{c^2}$$

$$v^2 \leq c^2 (1 - \sqrt{\frac{\rho_o}{\rho_P}})$$

Expression that represents the maximum limit of the velocity that a particle can reach and that depends on its physical properties.

For the quantity of motion. We will start from the relativistic quantity of motion.

$$p = p_o \gamma$$

$$\gamma = \frac{p}{p_o}$$

$$\gamma^4 = \frac{p^4}{p_o^4}$$

Where

$$\gamma^4 \leq \frac{\rho_P}{\rho_o}$$

$$\frac{p^4}{p_o^4} \leq \frac{\rho_P}{\rho_o}$$

$$p^4 \leq \frac{\rho_P p_o^4}{\rho_o}$$

$$p \leq p_o \sqrt[4]{\frac{\rho_P}{\rho_o}}$$

A particle with a quantity of motion is observed by an inertial observer. For any inertial observer it is satisfied that the maximum amount of motion it can measure satisfies the coordinate.

For the relativistic energy of the particle we start from.

$$p \leq p_o \sqrt[4]{\frac{\rho_P}{\rho_o}}$$

Squaring both members.

$$p^2 \leq p_o^2 \sqrt{\frac{\rho_P}{\rho_o}}$$

Multiplying both members by c^2 and adding $(m_o c^2)^2$

$$p^2 c^2 + (m_o c^2)^2 \leq p_o^2 c^2 \sqrt{\frac{\rho_P}{\rho_o}} + (m_o c^2)^2$$

Expression that coincides with the relativistic energy.

$$E^2 \leq p_o^2 c^2 \sqrt{\frac{\rho_P}{\rho_o}} + (m_o c^2)^2$$

A particle with a quantity of motion is observed by an arbitrary inertial observer. The expression obtained corresponds to the energy limit that can be measured by any inertial observer.

Implications and applications

Velocity limits on particles have found applications in more advanced sections of modern technology. For example, many sensors rely on relativistic velocities, since they use changes in travel time to detect moving bodies. New communication technologies have also been developed with quantum information by means of entangled photon pairs. In this type of transmission, special relativity ensures that the causality principle is not broken, since, although the states of two photons at the source are intertwined, each pair is sent to different receivers.

Although the speed of an object cannot exceed the speed of light, the simultaneity of events in space and time can be analyzed by quantum mechanics. These two theories seem to oppose each other. In quantum mechanics, the presence of pairs of entangled particles can cause a change of state of one particle to be immediately reflected in the other, regardless of the distance separating them. However, the causality principle states that no influence can propagate from one point to another faster than the speed of light, which harbors the idea that the variables are not clearly defined and that a change in one state is only reflected when both particles are measured.

Technological applications

Modern technologies benefit from speed limits. For example, sensors have been developed that use time-of-flight, allowing a signal to be sent and the time it takes to return to be measured. With a flight time of 0.000000001 seconds, the distance reached by the signal is 300 meters. This distance is sufficient to cover the depth of many bursts and explosions, allowing to detect the presence of the object in the area. Another example is found in telecommunications. In communications using fiberglass, the time it takes for signals to travel long distances is not much different from the time it would take for a signal traveling at the speed of light. At this speed, the principle of causality would be broken. However, causality in quantum mechanics seems to differ, as there are situations in which

it can be violated. In quantum mechanics there is a speed limit, but it cannot be used to send information.

Velocity limits are fundamental in physics, and also in technology. However, the origin of these limits is different from that shown in quantum theory. In this theory, velocity is an intrinsic property of a given particle; the fact that a particle has a velocity greater than c does not mean that it has violated any limit, but that it cannot be conceived of in the classical sense. Therefore, the study of velocity limits can be performed from two different approaches, the deterministic and the probabilistic.

Foundations of causality and causality in quantum mechanics

The study of causality in various physical fields highlights a surprising phenomenon: the causality of one of the theories, special relativity, is included in quantum mechanics, thanks to its relation to the velocities of certain particles. This means that, starting only from notions of special relativity in a quantum theory, one arrives without any assumption at an interpretation that gives meaning to a concept which, in this quantum theory, is meaningless. If these results are considered valid for quantum mechanics, as they are in the case of quantum causality, it could be argued that, under the framework of special relativity, the causality of quantum mechanics is located in a quantum form of special relativity.

Relativistic causality states that, given an event and an agent that alters the space-time of its environment, the effect can only be seen in a subsequent event for an observer moving at less than the speed of light. This allows the transmission of information only between events in which the first is in the future of the second and of which the distance is less than the time it will take to go from the first to the second multiplied by the speed of light. Causality in quantum mechanics must be a property of the theory rather than of its interpretation.

Critical discussion

The assumptions underlying the analysis of the velocity limits of a particle are extensive and, therefore, the discussion of their limitations is also extensive. The possibility that these limitations are not strictly physical, but derived from the standard model, is considered. The description of particles in the context of classical physics establishes acceleration and detection criteria that have made it possible to observe velocities close to c , although without reaching it.

In the context of special relativity, the concept of causal limit originates in relativistic causality, which implies that the structure of space-time makes it impossible for information to be transmitted faster than the speed of light. On the other hand, in quantum physics there appear interpretations in quantum physics that accept the transport of signals at speeds faster than the speed of light, albeit in the form of an associated probability.

The determination of such limits described in theories beyond the standard model has been a constant goal of research. The acceleration and detection methods for estimating velocity limits are diverse, as are the criteria for the accuracy of the estimates and their errors. The decision to apply a deterministic or probabilistic observation criterion must be related to the phenomenon under study.

Conclusions

In this article, it has been shown that the velocity limits for a particle depend on its type and the medium in which it moves, and that, by considering the equations of classical and quantum mechanics, it is possible to establish a precise boundary for the maximum allowed velocity, as well as to analyze the experimental and theoretical implications of these limits. The analyses have focused on particle velocities, although analogous concepts can be applied to other objects, such as wave propagation limits. While the speed of light in vacuum is universal and Lorentz-conserved, particle limits are specific to theoretical models, such as the standard model in particle physics. They have been analyzed on the basis of the principles that support them, the relationship with acceleration, detection and prediction methods, and the technological applications that derive from them.

The concept of causal limit is presented as the most general and derives from the causality implicit in special relativity. Since its origin, causality has conditioned the structure of space-time, and full respect for relativistic causality is the basis for its incorporation into quantum mechanics. In this framework, particle velocities that are much smaller than that of light are the foundation of a quantum particle model.

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