

Aprendiendo cálculo diferencial con procesamiento de imágenes digitales

Learning differential calculus with digital image processing

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Abstract:

In the daily routine of teaching, different problems usually arise concerning the teaching of mathematics, some of them are the lack of motivation of students, the myth that this subject is difficult, and even the lack of teaching strategies and digital tools that facilitate learning. This work presents a teaching strategy for differential calculus based on the application of the derivative concept to design and implement an edge detector, that represents a digital image processing method that consists of identifying the contours of objects in an image, based on the variation in brightness between neighbouring pixels. developed edge detector is then applied to clinical diagnosis on medical images, i.e. it highlights or makes more visible patterns characteristic of certain medical conditions. In this way, it is demonstrated that mathematics has applications in everyday life, which is intended to awaken in students the desire to learn it. Finally, the edge detector implementation is done in Matlab R2021a, since its image processing toolbox represents a user-friendly software for this purpose.

Keywords:

Education, learning mathematics, image processing, edge detection, differential calculus.

Resumen:

En la cotidianidad de la docencia, generalmente se presentan diferentes inconvenientes y/o problemas en la enseñanza de las matemáticas, algunos de ellos son la falta de motivación de los estudiantes, el mito acerca de que esta asignatura resulta ser difícil y tediosa, e incluso la falta de estrategias didácticas y herramientas digitales que faciliten su aprendizaje. El presente trabajo presenta una estrategia didáctica para la enseñanza del cálculo diferencial basado en la aplicación del concepto de derivada para diseñar e implementar un detector de bordes, que es una técnica de procesamiento de imágenes digitales que consiste en identificar los contornos de los objetos de una imagen, a partir de la variación en el brillo entre píxeles vecinos. Posteriormente, el detector de bordes desarrollado se aplica en el diagnóstico clínico al ser aplicada en imágenes médicas, esto es, resalta o hace más visibles patrones característicos de ciertos padecimientos médicos. De este modo, se demuestra que las matemáticas tienen aplicaciones en la vida cotidiana, con lo cual se desea despertar en los alumnos el deseo de aprenderlas. Finalmente, la implementación del detector de bordes se realiza en Matlab R2021a, dado que su toolbox de procesamiento de imágenes representa un software amigable para tal fin.

Palabras Clave:

Educación, aprendizaje de matemáticas, procesamiento de imágenes, detección de bordes, cálculo diferencial.

Introducción

The process of teaching and learning differential calculus has drawn global attention, largely due to the high failure rates, the difficulty many students face in understanding the subject, increased dropout rates, and a growing

tendency to choose academic programs that do not include it in their curricula [1]. In response to this situation, various studies have addressed the issue and proposed different methodologies to enhance student learning and academic performance[2]. In this context, we propose a teaching strategy to help students better understand the

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concept of the derivative, through the design of an edge detector and its application to medical images for disease identification tasks. In this way, the present work pursues the following objectives:

- To design an innovative and accessible teaching method that uses medical image processing to help students better understand the concept of the derivative.
- To implement the proposed strategy using Matlab2021a, enabling a practical and interactive learning experience.
- To design an innovative and accessible teaching method using medical image processing to promote a better understanding of the concept of the derivative.

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A digital image is a two-dimensional representation of a scene through a numerical matrix (Figure 1).

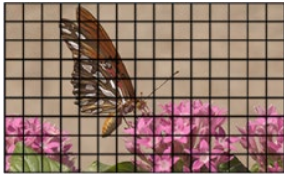


Figure 1. Digital image.

Mathematically, it is expressed as a function:

$$I(x,y) \dots\dots\dots(1)$$

There are several ways to explain how colors are formed in digital images, and the RGB model is one of the most commonly used [3]. It works on the idea that any color can be created by combining three primary colors (Red–Green–Blue), as illustrated in Figure. 2.



Figure 2. RGB color model.

In this way, the brightness or luminous intensity of a pixel is represented by a vector that includes the three-color components, each with a value within the range [0, 255], as shown in Figure 3.

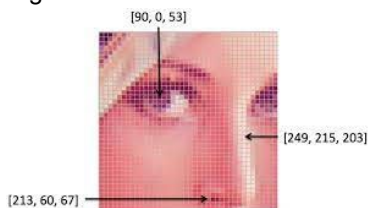


Figure 3. Pixel brightness intensity.

Sometimes, when working with images, it is more practical to convert a color image to grayscale. This is because grayscale images assign a single brightness level to each pixel, making processing simpler by representing each point as a shade of gray (see Figure 4).

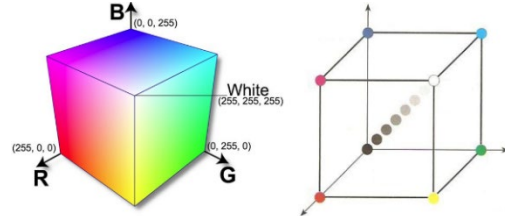


Figure 4. Grayscale.

This conversion is carried out using the following expression:

$$I_{gray}(x,y) = 0.2989R + 0.5870G + 0.1140B \dots (2)$$

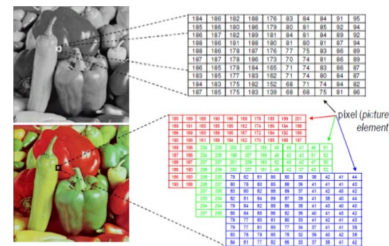


Figure 5. RGB to grayscale conversion.

Edges in an image are understood as the set of pixels where there is a noticeable change in their gray level compared to neighboring pixels (Figure 6) [4].

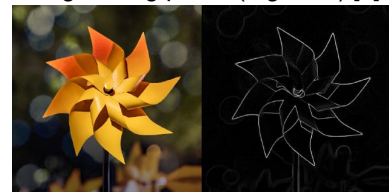


Figure 6. Detected edges in an image.

There are two types of edges: those that correspond to the contours of objects (strong edges), and those that result from texture details (weak edges), as illustrated in Figure 7.



Figure 7. Types of edges in an image.

Edges are identified by computing image derivatives in both the x and y directions. The derivative is defined as follows (see Figure 8).

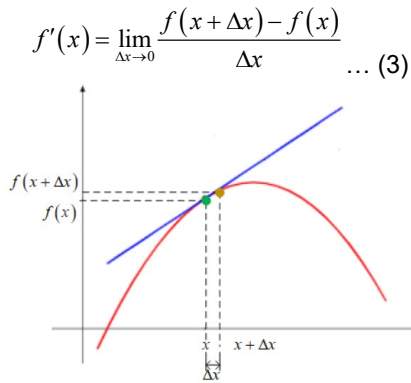


Figure 8. A geometric interpretation of the derivative.

An image is considered a discrete signal, as illustrated in Figure 8. Therefore $\Delta x = 1$.

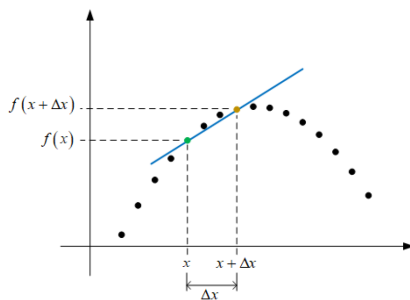


Figure 9. Discrete function.

Therefore, Eq. (1) is reduced to:

$$f'(x) = f(x+1) - f(x) \quad \dots\dots\dots (4)$$

This means that the derivatives of a digital image can be approximated by simply subtracting a pixel from its neighbor. By combining Equations (4) and (1), the image derivatives can be calculated as follows:

$$I_x = I(x+1, y) - I(x, y) \quad \dots\dots\dots (5)$$

$$I_y = I(x, y+1) - I(x, y) \quad \dots\dots\dots (6)$$

Both derivatives are combined into a vector known as the image gradient [5]:

$$\nabla I = [I_x, I_y]^T \quad \dots\dots\dots (7)$$

The magnitude of this vector determines which pixels are considered edges in the image. To achieve this, a threshold must be defined, so that pixels with a higher gradient magnitude are identified as edges. Using this approach, results such as the one shown in Figure 10 can be obtained.



Figure 10. Edge detection.

This methodology can be applied to the identification and monitoring of certain medical conditions. For instance, in mammographic images, texture plays a key role in determining whether tissue is affected by cancer, or in detecting calcifications and cysts (see Figure 11).

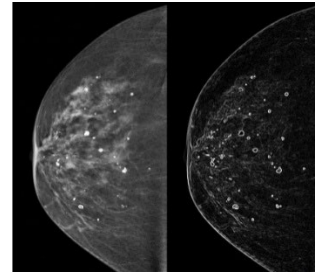


Figure 11. Mammogram image processing.

Edge detection can also be applied to angiograms for their identification. It is important to note that this type of image is typically low in resolution and contrast, which makes interpretation more challenging, as illustrated in Figure 12.

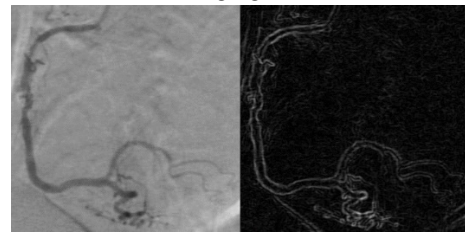


Figure 12. Angiogram image processing.

Conclusion

The proposed methodology for teaching the concept of the derivative through medical image processing offers an innovative approach to addressing the lack of interest and motivation often observed among higher education students in learning mathematics.

Referencias

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