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Gliders in Rule 110

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The existence of several periodic structures (known as *gliders*) in the evolution space of the one-dimensional cellular automaton Rule 110, has important lines of investigation in cellular automata theory such as: complex behavior, universal computation and self-reproduction. We investigate the types of gliders, their properties and production through collisions in Rule 110 and their representation by tiles.

1. INTRODUCTION

The one-dimensional binary cellular automaton numbered Rule 110 in Stephen Wolfram's system of identification [47] has been an object of special attention due to the structures or gliders[1] which have been observed in evolution samples from random initial conditions. It has even

[1] Glider is a periodic structure moving through time. This term is taken from the literature of the cellular automaton proposed by John Horton Conway "The Game of Life" [15]. Good references on line are: "LIFEPAGE" http://members.aol.com/life1ine/life/lifepage.htm and, "LifeInfo" http://www.pentadecathlon.com/LifeInfo/LifeInfo.html

been suggested that Rule 110 belongs to the exceptional class IV of automata whose chaotic aspects are mixed with regular behaviors; but in this case the background where the chaotic behavior occurs is textured rather than quiescent, a tacit assumption in the original classification.

Whatever the merits of this classification, Rule 110 was awarded its own appendix (Table 15) in reference [47]. It contains specimens of evolution including a list of thirteen gliders compiled by Doug Lind and also presents the conjecture thatthe rule could be universal.

There is little published literature about Rule 110; an example is a statistical study [30] done by Wentian Li and Mats Nordahl around 1992. This paper studies the transitional role of Rule 110 and its relation with class IV rules sitting between Wolfram's classes II and III. This study would seem to be reflected in an approach to equilibrium statistics, via a power law rather than exponentially.

Matthew Cook wrote an eight page introduction[2] [10] listing gliders from *A* through *H* and a glider gun.[3] The list of Cook shows new gliders which do not appear in the list of Lind, gliders with rare extensions and a pair of gliders of complicated construction. In that document Cook makes a comparison between Rule 110 and Life, finding some similarities and suggesting that Rule 110 may be called "LeftLife."

Looking at the rule itself, one can see an ubiquitous background texture which Cook calls "ether" although it is just one of many regular stable lattices able to be formed by the evolution rule.

This approach is suggested by observing that the basic entities in the lattices, the unit cells, induce the formation of upside-down isosceles right triangles of varying sizes. The significance of Rule 110 could be in taking recognizably distinct tiles tobe assembled, and now the evolution can be studied as a tiling problem [18], in the sense of Hao Wang [45]. It might even be possible to see fitting elements of one lattice into another as an instance of Emil L. Post's correspondence principle, which would establish the computational complexity of the evolution rule [33].

Harold V. McIntosh raises the question where Rule 110 is a problem to cover the evolution space with triangles of different sizes [34]. It is convenient to use the symbol T_n to represent one of these triangles, the

[2] The information at the moment is not available in Internet due to legal problems, a good reference discussing this problem may be consulted in [16]. A brief historical introduction and development on Rule 110 is displayed in [21] available from http://www.rule110.org/download/

[3] Through this paper we use the glider classification proposed by Cook in [10].

number n indicating itssize or the number of cells with state 0 just below the top margin. The appearance of these triangles suggests the analysis of the plane generated by the evolution of Rule 110 as a two dimensional shift of finite type.

The most important result both in the study of Rule 110 and in cellular automata theory in the last twenty years, is well represented by the demonstration made by Cook that Rule 110 is universal.[4]

The demonstration is realized simulating a cyclic tag system [12] and [48]; with well-defined blocks of gliders by means of collisions. The simulation in the evolution space of Rule 110 is really complicated and several details must be taken into account to have a good construction as can be seen in [35] and [25].

The relevance of the demonstration realized by Cook is to reduce the neighborhood, state set and dimensionality to the possible minimum. Then we can say that Cook has the last reduction with the simplest cellular automaton able to produce universal computation [12] and [48]. Rule 110 is a one-dimensional cellular automaton with two states and a linear three cells neighborhood. The transition function simultaneously evaluates a central cell with regard to its left and right neighbors.

Gliders in the cyclic tag system are useful to represent process collision-based computing. In the model of Conway, universality is demonstrated simulating a register machine through logic gates [6], constructing the system with gliders and glider guns. At the present time Paul Rendell has implemented a complicated Universal Turing Machine in Life [2].

Rule 110 is characterized by having a wide variety of gliders in its evolution space. The first question is to determine if the list of gliders in Rule 110 is complete, a problem up to now open. The second question is to know the objects or devices that can be constructed. For instance at the present time new structures and more complicated systems are being found and constructed in Life as we see in [2] and [17], probably they could be found in Rule 110 too.

Rule 110 is able to produce a large variety of objects that can be useful for the construction of other systems in their evolution space. However, in Rule 110 it is not possible to find still life objects, these stable structures have a particular interest in Life as Cook shows in [11].

[4] Part of its results were published in [48], a published version appeared in Complex Systems for 2004 [12]

Several of our results were discussed later with Cook, until November of 2002. The original element in this investigation is the characterization of each glider and the language established to describe collisions in the evolution space. We apply de Bruijn and cycle diagrams for describing gliders and other periodic constructions, their representation using tiles and we use subset diagrams to determine configurations without ancestors.

This description is realized through regular expressions [26] derived from the de Bruijn diagram [32]. A problem is that de Bruijn diagrams grow exponentially. In order to solve this problem, we calculate all the regular expressions by means of the phases f_i_1 [22].

The procedure consists in concatenating the regular expressions and constructing initial conditions for a particular intention. We applied the phases to construct interesting initial conditions as production of gliders, groups of gliders, meta-gliders, and Rule 110 objects through collisions.

Then we determine the set of all the f_i_1 phases for each glider, including the glider gun. Finally this procedure is taken into the computational system OSXLCAU21.[5]

The constructions with phases[6] depicted in this article were made with the OSXLCAU21 system and other important calculations were generated with the NXLCAU21 system.[7]

2. GLIDERS IN RULE 110

A cellular automaton is a discrete dynamical system evolving through time. Rule 110 is a binary one-dimensional cellular automaton, number 110 talks about the decimal notation of the evolution rule which is the binary sequence 01110110.

The automaton is defined by a finite one-dimensional array where each one of its elements takes the value 0 or 1 from the state set, this array represents the initial configuration of the system. A neighborhood is

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[5] OSXLCAU21 system for OPENSTEP, Mac OS X and Windows. Application and code available in: http://delta.cs.cinvestav.mx/˜mcintosh/comun/s2001/s2001.html, and http://www.rule110.org/download/

[6] Each production offers a detailed expression represented with phases to reproduce every example in Appendix A, although it does not mean that it is the only way as they can be produced

[7] Set of programs NXLCAU developed for NeXTSTEP and OPENSTEP. Application and code available in: http://delta.cs.cinvestav.mx/˜mcintosh/oldweb/software.html

formed by three cells, a central element,a neighbor to the right and another into the left. Depending on the values in cells of the neighborhood, a new value is defined for the central cell in the next generation.

The transition function evaluates synchronously each neighborhood to calculate the new configuration. Thus the transition function transforms the neighborhoods 001, 010, 011, 101 and 011 into state 1 and the neighborhoods 000, 100 and 111 into state 0.

Figure 1 shows that Rule 110 has a uniform behavior represented by ether, periodic behaviors represented by gliders and chaotic behaviors represented by the unstable regions. The interesting thing in Rule 110 is represented by collisions, because collisions among gliders can originate the three previous behaviors.

Now we present the glider classification in Rule 110 proposed by Cook. We introduce an extended glider gun which was found on searching a collision producing the glider gun.[8] For a given glider, the superscript n means that it has extensions of arbitrary length as Figure 2 shows, where n is a positive integer.

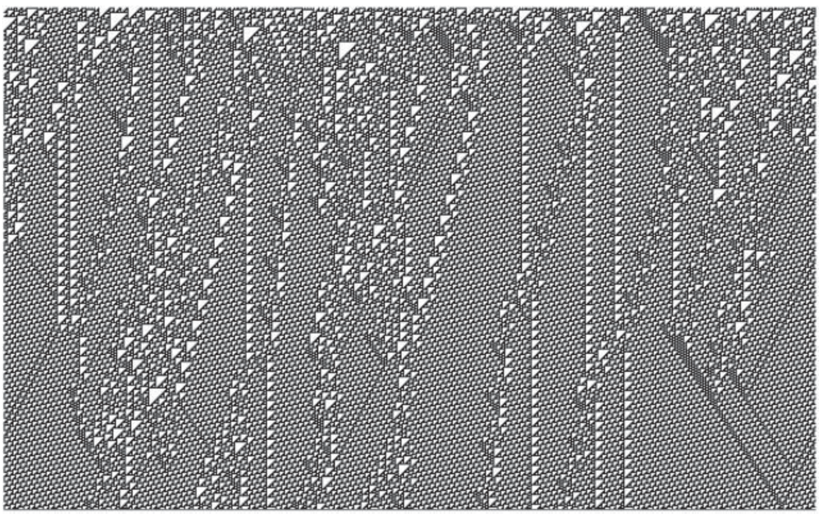

FIGURE 1
Random evolution in Rule 110.

[8] Cook also knew about the extended glider gun although he did not include it in his classification (personal communication)

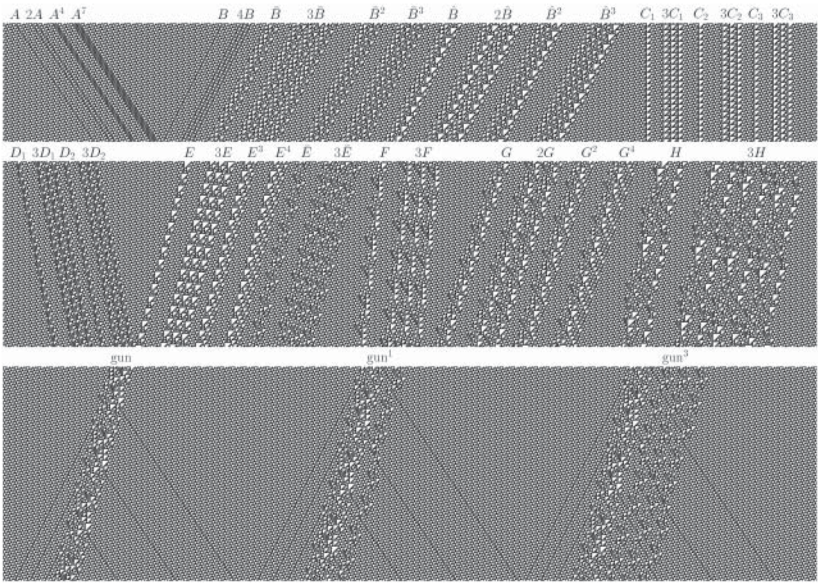

FIGURE 2
Classification of gliders in Rule 110.

$\bar{B}^{n>1}$, $\hat{B}^{n>0}$, H gliders and the glider gun were discovered by Cook. A and D gliders are the only ones going from left to right, C gliders do not move and the rest of the gliders shift to the left.

Gliders in Rule 110 offer a wide variety of combinations, we can have groups or packages of them, hence their representation can be a problem. For example, how can we to represent two join $\bar{B}$ gliders?, as $2\bar{B}$ or $\bar{B}^2$?. In this paper we shall follow the representation proposed by Cook where A^n depicts n joined A gliders and $\bar{B}^n$ is a n-extended $\bar{B}$ glider.

The $\bar{B}^n$ and $\hat{B}^n$ gliders have complicated and difficult extensions. Some gliders and tiles may cover the evolution space without the intervention of tile T_3 but others gliders dont. Besides, the number of ways in which every glider can be joined to cover the evolution space is different for each glider. G glider is the unique one that can cover the evolution space in eight different ways. Tiles T_1, T_2, T_3 and T_4 also cover the evolution space by themselves [23].

Classification of tiles is shown in Figure 3 [34]. Tile T_3^β represent ether, but we shall only write T_3 forward.

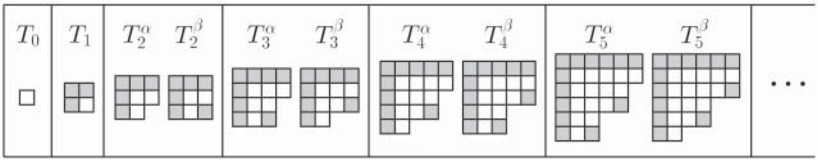

FIGURE 3
Two types of tiles in Rule 110: alpha and beta.

For example, the H glider is formed by 263 tiles, where 31 are T_3 tiles. This glider cannot cover the evolution space by itself like A glider, because it needs additional T_3 tiles for covering empty regions.

The margins of the gliders follow a construction like the lines of Bresenham's algorithm [8]. They can be represented as a hexagonal lattice with 3 (or 6) principal directions, and margins of gliders tend to follow them.

We have gliders exhibiting internal collisions. Figure 4 shows in detail each internal collision in a H glider and the glider gun. The internal

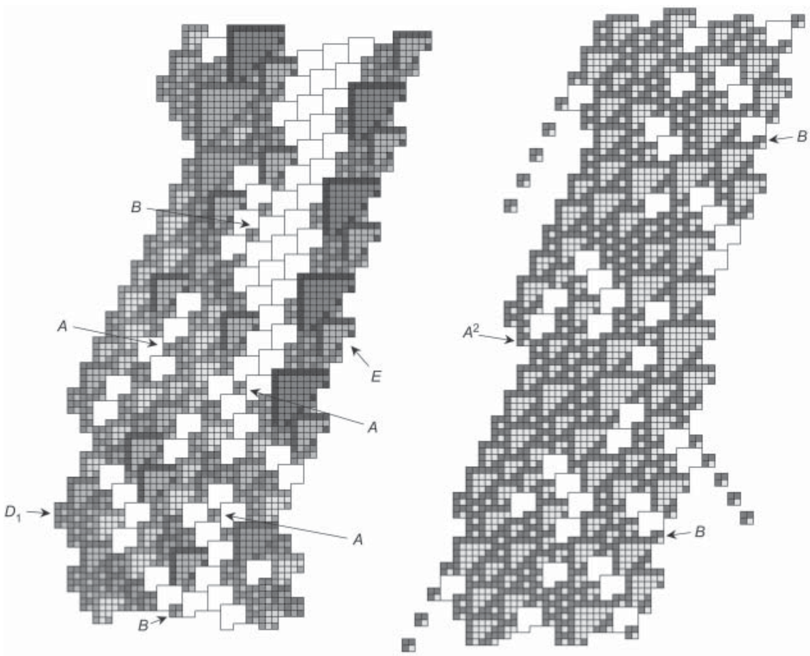

FIGURE 4
H glider and glider gun composite by internal collisions.

construction of a H glider is well-defined by the existence of an E glider in its right margin; three collisions with two A gliders and the T_2 tile allow a periodic return.

In the case of the glider gun, several fragments of other gliders can be identified as E, $\bar{E}$, F and G gliders. It is important to notice that the superior margin of $\bar{E}$ glider is the same that the F glider or fragments of left marginsin G, H gliders and the glider gun.

In the figure, empty T_3 tiles are introduced to clarify the parts where the H glider and the glider gun have their internal collisions. H glider has the largest tile with a T_9, whereas E, G gliders and the glider gun have T_8 tiles. If there are more gliders, a relevant question is to know the largest T_n tile inside a glider.

In the structure of the glider gun it is more difficult to identify natural collisions between gliders because the interior of this glider is mostly formed by decompositions with a fast interaction. In the case of $\bar{E}$, F, G and H gliders the internal collisions are clearer.

Glider gun in Rule 110 just as in Life determines an unlimited constant growth in the evolution space.

Finally gliders can be divided in two classes: *natural* and *composite*. The natural ones are those formed without some internal interactions and composite gliders are formed by the interaction of at least a natural pair of gliders. This relationis in Table 1 (letters ‘cd’ mean a chaotic decomposition).

In this context we can discuss about the existence of more gliders in Rule 110, because we have structures fulfilling conditions of collisions that help to form new gliders conserving their forms.

Gliders in Rule 110 have important restrictions determined by the T_3 tile. Characterizing the evolution space for Rule 110 by means of tiles allows to use what we call “phase analysis” [22] (described in section 3.2).

TABLE 1
Two classes of gliders in Rule 110.

natural gliders	composite gliders
A	$\bar{E} = A \to B, A^2 \to$ cd
B	$F = A \to B, B \to$ cd, $A^2 \to C_2$
$\bar{B}$	$G = A^2 \to$ cd
$\hat{B}$	$H = B \to$ cd, $A \to$ cd, $A \to E, D_1 \to$ cd
C_S	glider gun = $A^2 \to$ cd
D_S	
E	

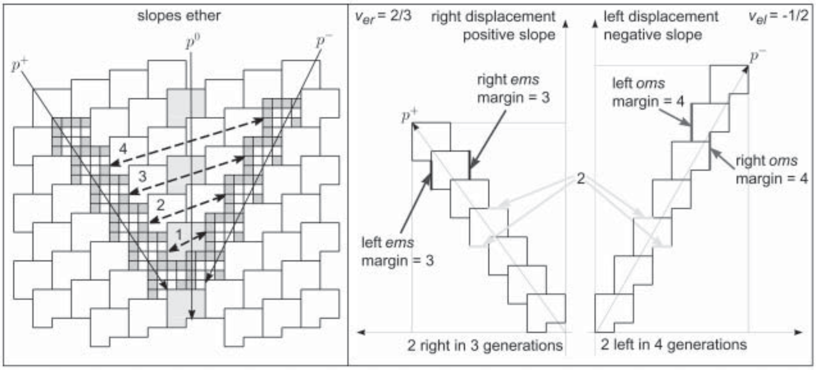

FIGURE 5
Three types of slopes produced by ether.

Ether represents three types of slopes as Figure 5 show: positive slope p^+, negative slope p^- and null slope p^0. They determine the maximum positive and negative velocity in Rule 110 for the periodic structures in the evolution space.

Note that there are *left* and *right* margins in both slopes for each T_3 tile. If the height of the margin is *even* (with 3 cells) the margin is represented by 'ems' positive slope. In the opposite case if the height of the margin is it *odd* (with 4 cells) the margin is represented by 'oms' negative slope.

Bresenham's algorithm has a fixed increment in both axes for tracing a 45 degrees straight line. The slope of ether (p^+, p^- or p^0) has fixed increment in both axes as well. The increment in the horizontal axis is of 2 cells for any slope and the increment in the vertical axis is of 3 cells for p^- and 4 cells for p^+. Thus a line of T_3 tiles is useful to identify the edge of a glider as we can see in Figure 5.

The slopes of T_3 tile determine the advances and backwards of each glider in Rule 110. Thus it is possible to project constructions of gliders or new periodic patterns formed by different tiles through their slopes.

The slopes of ether establish both the limits and the contact points for any structure in Rule 110. A *contact point* is a part of the left or right margin where a glider can take a hit with any fixed or in-movement structure. A *non-contact* point is a part of the left or right margin where a glider cannot take a hit [40]. Contact points are determined by the number of odd margins *oms* for p^- or even margins *ems* for p^+.

If p^+ has a right shift of 2 cells in 3 generations, then the ether speed with this slope is $v_{er} = 2/3$. If p^- has a left shift of 2 cells in 4 generations, then the ether speed with this slope is $v_{el} = -1/2$.

For p^+, if we have *n ems* margins (to the right), then there must exist *n ems* margins (to the left). For p^-, if there are *n oms* margins (to the left) then there must exist *n oms* margins (to the right). In other words the existenceof a contact point in one margin implies the existence of other non-contact points in the opposite margin. Then both margins have a bijective correspondence in every glider [22].

All periodic structure in the evolution space of Rule 110 advances +2 cells and backs down –2 cells. Then all structure with p^+ has a velocity $v \leq v_{er}$. In the other case for p^- has a velocity $v \leq |v_{el}|$, where v represents the velocity of a glider in Rule 110. All structure with p^+ advances with increments of v_{er} and backs down with decrements of v_{el}. In the other case every structure with p^- advances with increments of v_{el} and backs down with decrements of v_{er}.

Each structure with p^+ can be affected by another structure moving in the opposite sense just if the first has at least one *oms* margin and the second has at least one *ems* margin. In the opposite case every structure with p^- can be affected by another structure in opposite sense just if the first has at least one *ems* margin and the second has at least one *oms* margin.

Margins determine another important property in gliders: the non-contact and contact points. That is, regions where a glider may collide or not with others gliders.

Figure 6 illustrates both the contact and non-contact points in the G glider. In the right side the *ems* margins are contact points which interact with other glider, but the *oms* margins are points where there is no collision with other glider.

Margins are not only represented in gliders but also in non-periodic regions as Figure 7 illustrates. In these regions the bijective correspondence between margins is not conserved and their number depends on the evolution.

Finally with the properties of the margins *ems* and *oms* we determined the following equations.

Let $\mathcal{G}$ be the set gliders in Rule 110, for any $g \in \mathcal{G}$ the displacement of g can be represented with the following equation:

$$d_g = (2 * oms) - (2 * ems). \tag{1}$$

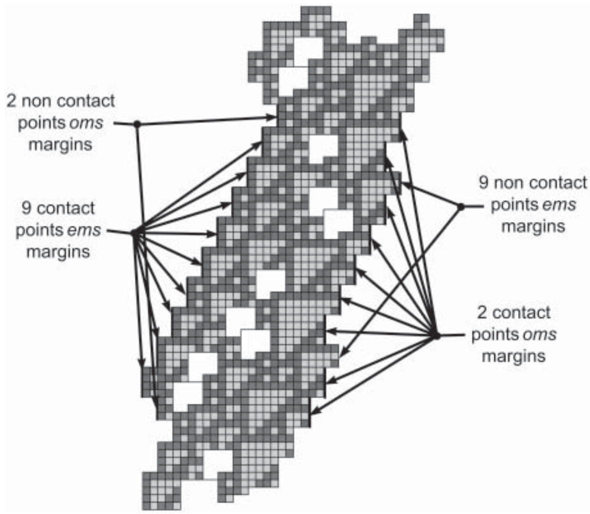

FIGURE 6
Margins and contact points in *G* glider.

All periodic structure has a period defined by the amount of *oms* and *ems* margins. Therefore the period of the *g* glider is defined by:

$$p_g = (3 * ems) + (4 * oms) \tag{2}$$

finally the *g* glider in Rule 110 has a speed given by:

$$v_g = \frac{(2 * ems) - (2 * oms)}{(3 * ems) + (4 * oms)}. \tag{3}$$

Collisions between gliders have a maximum level determined by the number of margins *oms* and *ems*. A glider with *oms* contact points and other with *ems* contact points hold that:

$$c \leq ems * oms \tag{4}$$

where *c* represents the maximum number of collisions between both gliders. Nevertheless for some gliders with non-contact and contact points the maximum level is not fulfilled. Taking the equality, the number of collisions between two gliders g_i and g_j, where $i \neq j$, is represented by the following equation:

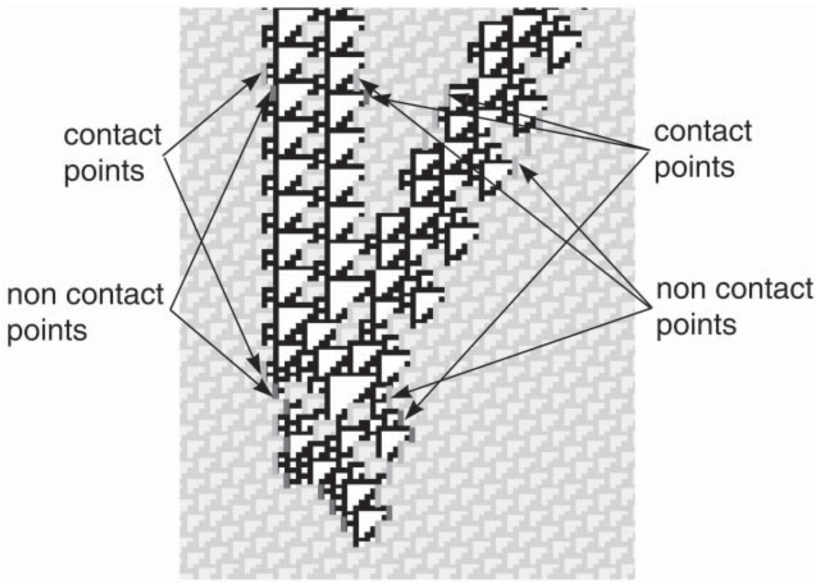

FIGURE 7
Margins and contact points in nonperiodic structures.

$$c = |(ems_{g_i} * oms_{g_j}) - (ems_{g_j} * oms_{g_i})|. \tag{5}$$

Each property is detailed for every glider including the glider gun, data are displayed in Table 2.

The first column indicates a particular glider, the structures e_r and e_l represent ether with slope p^+ and p^- respectively. The four following columns indicate the number of margins in each glider, the first two columns show the even and the odd margins in the left side and the other two columns the same information for the right side of the glider.

Column v_g indicates the speed of each glider, calculated by dividing the shift of the glider between its period. Column *width* indicates the periodic length of each glider. The last column indicates the gliders which can cover the evolution space, T represents a total covering without any T_3 tile and P represents a partial covering with at least one T_3 tile.

3. DETERMINING GLIDERS IN RULE 110

Now we discuss the tools used to calculate gliders in Rule 110. Several interesting aspects are found in the analysis.

TABLE 2
Margins and properties of gliders to Rule 110

structure	margins left - right				v_g	width	covering
	ems	*oms*	*ems*	*oms*			
e_r	.	1	.	1	$2/3 \approx 0.666$	14	T
e_l	1	.	1	.	$-1/2 = -0.5$	14	T
A	.	1	.	1	$2/3 \approx 0.666$	6	T
B	1	.	1	.	$-2/4 = -0.5$	8	P
$\bar{B}^n$	3	.	3	.	$-6/12 = -0.5$	22	T
$\hat{B}^n$	3	.	3	.	$-6/12 = -0.5$	39	T
C_1	1	1	1	1	$0/7 = 0$	9–23	P
C_2	1	1	1	1	$0/7 = 0$	17	P
C_3	1	1	1	1	$0/7 = 0$	11	P
D_1	1	2	1	2	$2/10 = 0.2$	11–25	P
D_2	1	2	1	2	$2/10 = 0.2$	19	P
E^n	3	1	3	1	$-4/15 \approx -0.266$	19	P
$\bar{E}$	6	2	6	2	$-8/30 \approx -0.266$	21	P
F	6	4	6	4	$-4/36 \approx -0.111$	15–29	P
G^n	9	2	9	2	$-14/42 \approx -0.333$	24–38	P
H	17	8	17	8	$-18/92 \approx -0.195$	39–53	P
glider gun	15	5	15	5	$-20/77 \approx -0.259$	27–55	P

3.1 de Bruijn diagram

The main thing with a graph (meaning, digraph, since the distinction is important in this application) is that loops may be isolated, connected in one direction but not in the other, or mutually connected.

With regard of de Bruijn diagrams [32], the first alternative would mean that there is a simple pattern, persistent or shifting as the case may be, but essentially unique, not admitting any variation. For example, superluminal patterns generallyhave this form since causality is not operating.

The nodes of the de Bruijn diagram are sequences of symbols from some alphabet, just as regular expressions are. They can even be sequences of nodes from a specific graph. The links of the diagram describe how such sequences may overlap. Different degreesof overlap lead to different diagrams, the simplest of which overlap according to the gain or loss of a single initial or terminal symbol.

When the symbols are consecutive integers they can be treated as elements of a ring or perhaps a finite field; the ease of discussing their properties arithmetically or algebraically makes the choice eminently worthwhile.

The unilateral connections correspond to fuses, which is an irreversible change of pattern which may be either static or shifting. Many configurations for Rule 110 have this form, including most of the speed zero shifts, and in particular the C gliders,which can abut on uniform quiescence, or vacuum.

Gliders depend on a loop generating the ether which has another connection to itself constituting the glider. The ether loop could be an autolink to the quiescent state, but things are different in Rule 110. There might possibly be several handles, signifying distinct forms of gliders or different phases in the evolution of a single glider all moving at the same velocity.

A still more complicated combination has the glider off in a loop of its own, but still having mutual connections to the ether loop. That is the arrangement with respect to the extensible gliders, and can be used to determine admissable spacings, closest approaches, and so on. And it is a property of regular expressions, that if the glider is one extensible, it is multiply extensible.

Underlying the existence of the tiling is the fact that Rule 110 has semipermeable membranes. That is just a fancy way of saying that the sequence $x10$ always generates 1 (almost always the "semi" comes from $111 \rightarrow 0$); more pertinent is that $x10^n$ generates $10^{(n-1)}$ which is another way of characterizing the triangles. Membranes are traceable to configurations in the de Bruijn diagram. It remains to be seen how directly this membrane affects the analysis of Rule 110, even though it is an integral part of the characterization of Rule 110 by tiling.

The reason for mentioning this is that it has been know that some rules have membranes bounding macrocells [31], within which evolution has to seek a cycle. But not all membranes are permanent, leading to the conjecture that their dissolution might be programmed. This is a idea which has probably never been followed up, but Rule 110 may actually be an instance which fits the pattern, since the evolution depends on the persistence of the left margin in the triangles, and the way in which it eventually breaks up.

Figure 8 illustrates all the evolutions derived from the de Bruijn diagram up to 10 generations. When the two numbers coincide the diagram consists exclusively of loops, but not necessarily one single loop. Since zero is a quiescentstate, entries of the form (1,1) indicate that it is the only configuration meeting the shifting requirement. In particular, there are no still life (except for zero).

Points of interest in the figure, actually some of Cook's gliders, are the entries at (2,3) A-gliders, at (–2,4) B-gliders, at (0,7) C-gliders, and at

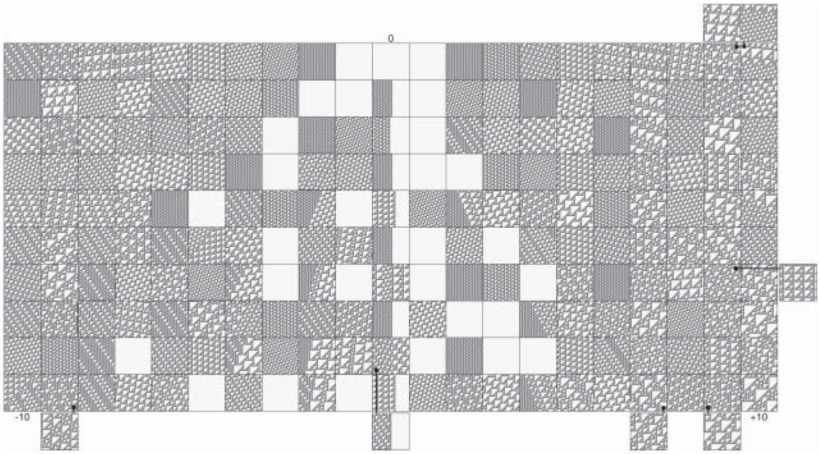

FIGURE 8
Patterns calculated with de Bruijn diagrams for 10 generations.

(2,10) D-gliders. The symbol (x, y) indicates a shift of x, negative to the left in y generations.

Cook's gliders are found in different phases. For example in (–2,3) the A glider completely covers the evolution space, in (–6,2) a group of A^2 is interchanged with a T_3 tile, in (–10,1) it is an A^3, in (–6,6) it is an A^4, in (–6,8) it is an A^5, in (–8,7) it is an A^6, in (–8,9) it is and A^7 and so on. But also we can see configurations grouping T_3 tiles in different groups of A gliders as it can be seen in coordinates (–8,6), (–8,10), (–10,8), (4,6) and (6,9).

In order to know the number of ways in which a particular glider or a group of them can cover the evolution space, is solved as well by the de Bruijn diagram. A clear example is with the D and C gliders, in coordinates (–9,1) and (–9,10) we have a D_2 glider but a careful observation shows that both evolutions are different by a T_3 tile, these are the only ways in which a D_2 glider can cover the evolution space.

In the same figure we can see as well how the different tiles T_1, T_2, T_3 and T_4 can cover the evolution space. We can find extensions for E and B gliders or configurations working as glider guns in (–10,10) and (0,9), in the secondcase the periodic configuration of the right is disturbed producing A gliders and these are cancelled with B gliders forming the periodic configuration on the left.

In the same coordinate (0,9) the de Bruijn diagram calculates two periodic regions with different cycles. The second configuration illustrates how a set of A gliders is contained by a shift defined from the initial configuration, representing the opposite leg of a very large tile. Calculating all the de Bruijn diagrams we found repeated evolutions in several coordinates, we do not put all the repetitions for lack of space and because they are the same evolution. In the case where two evolutions are repeated in the same place and they are not found in other coordinates, then they are indexed on one side.

Another important point is that de Bruijn diagrams can find periodic configurations constructed by large tiles. For example in coordinate (10,10) we have that a T_{11} tile may cover the evolution space with help of other additional tiles, in this way we can find similar evolutions for the tiles T_{10}, T_9, T_8, T_7 among others. Up to 10 generations the de Bruijn diagram define them.

Figure 9 illustrates the de Bruijn diagram calculating shifts of nine cells to the right for nine generations. In the diagram we found four

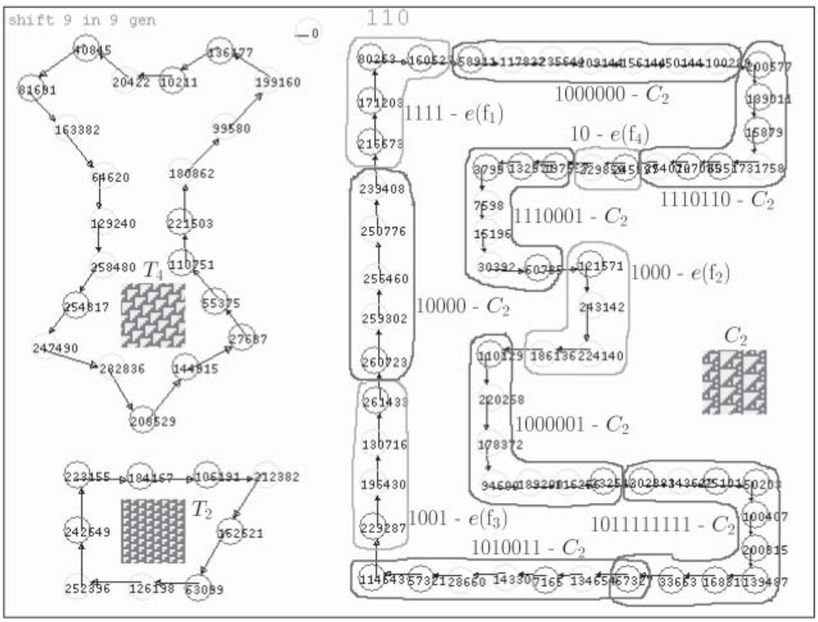

FIGURE 9
de Bruijn diagram calculating a C_2 glider.

cycles. First cycle is vertex zero stable state. Second cycle determines the initial conditionsto cover the evolution space with a T_4 tile. Third cycle shows the same behavior but with one T_2 tile of two possibilities in Rule 110.

Fourth cycle is more interesting, we can see each sequence determining a C_2 glider. All the cycle produces C_2 gliders united with spaces equal to a T_3 tile. So these C_2 have two (or more) periods.

Finally the de Bruijn diagram can determine the properties for all the periodic structures in Rule 110 and the great amount of information so produced can be widely discussed.

Rule 110 has several interesting aspects in cellular automata theory, another one is to determine configurations that belong to the Garden of Eden [38,3]. These are configurations that cannot be constructed by the evolution rule through time, for instance the expressions (101010)* and (01010)* cannot be constructed by Rule 110.

From the de Bruijn diagram we derive the subset diagram using the power set [32]. We obtain these sequences in the Garden of Eden following the routes from the maximum set (vertex 15) to the minimum set (vertex 0). These is a regular expression describing all of them.

Figure 10 illustrates the subset diagram for Rule 110, the paths with gray color represent the state zero and those with black color show the state one.

We can find several sequences in the Garden of Eden following these routes. For example the sequence (011010111010)* is another configuration which cannot be generated in the evolution of Rule 110, this sequence in the initial configuration producesgroups of A^2 gliders in

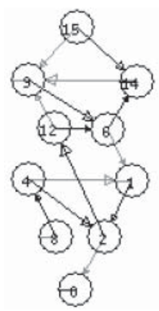

FIGURE 10
Subset diagram in Rule 110.

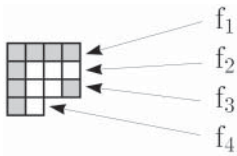

FIGURE 11
T_3 tile determinate ether in four phases.

intervals of two T_3 tiles. With the subset diagram we can see that all configuration in the Garden of Eden must finish with the sequence 10.

3.2 Phases in Rule 110

The de Bruijn diagrams can determine all the regular expressions for each glider in Rule 110. Nevertheless at the present time it is complicated to calculate diagrams for periods larger than ten generations. In order to solve this problem we propose a procedure called phases [22].

We return to the T_3 tile (Figure 11) for calculating all the regular expressions derived from the gliders in Rule 110. A phase represents an periodic sequence aligned to the tile determining ether.

T_3 tile has four phases (or sequences): $1111 = f_1$, $1000 = f_2$, $1001 = f_3$ and $10 = f_4$. The concatenation of them forms a periodic phase of ether: $11111000100110 = f_1f_2f_3f_4$.

Ether has a shift of 14 cells to the right in 7 generations. Thus we calculate all the regular expressions determined by ether: $11111000100110 = e(f_1\_1)$, $10001001101111 = e(f_2\_2)$, $10011011111000 = e(f_3\_3)$ and $10111110001001 = e(f_4\_4)$. The evolution space can be covered with ether from the initial configuration by anyone of them.[9]

Each phase is a permutation of the first, this implies that a phase is enough to establish a horizontal measurement. Nevertheless it is possible to mix all the phases for some particular construction, although this is not useful o simplify the analysis.

We align the measurement to a phase f_i_i, each phase has four levels due to the phases of the T_3 tile as Figure 12 shows. Then there are four phases f_i and each may be aligned i times generating all the possible phases (Table 3), where $1 \leq i \leq 4$.

[9] For example the proposed classification of gliders by Lind in its table 15 (Appendices page 577 [47], or http://www.stephenwolfram.com/publications/articles/ca/86-caappendix/16/text.html) corresponds to one of our sets of phases then ether is determined by phase $e(f_3\_3)$

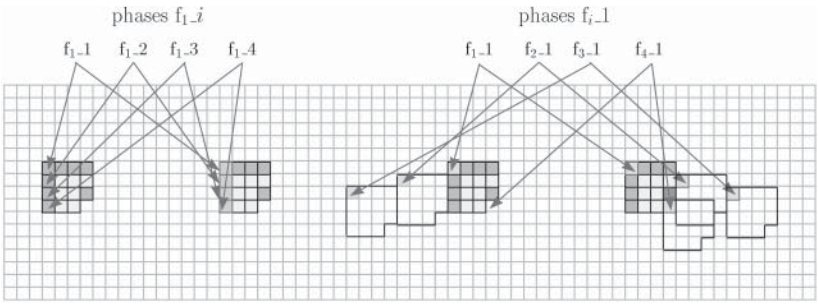

FIGURE 12
Phases f_i_i in Rule 110.

TABLE 3
Four sets of phases in Rule 110.

phases level one (Ph_1)	$\rightarrow$	f_1_1, f_2_1, f_3_1, f_4_1
phases level two (Ph_2)	$\rightarrow$	f_1_2, f_2_2, f_3_2, f_4_2
phases level three (Ph_3)	$\rightarrow$	f_1_3, f_2_3, f_3_3, f_4_3
phases level four (Ph_4)	$\rightarrow$	f_1_4, f_2_4, f_3_4, f_4_4

Let us remember that the phases are regular expressions or sequences of finite length in the de Bruijn diagram. It is important to indicate that an alignment of a phase determines one set of regular expressions, and another alignment determines a distinctset of them. This way we have four possible disjoint sets: Ph_1 (phases one), Ph_2 (phases two), Ph_3 (phases three) and Ph_4 (phases four).

Note: the combination of elements from different sets does not hold the properties of regular expressions, therefore they cannot operate under the rules of juxtaposition, *, and $\vee$. Thus the property of regular expressions is only conserved in the elements of each set.

Actually the de Bruijn diagrams can generate any periodic sequence in the evolution space of any cellular automata. But it has two problems: the diagram grows exponentially and the information must be extracted from a complicated graph.

The way to calculate both all the set of regular expressions for all gliders of Rule 110 and their alignment is by means of phases f_i_1. In order to determine the phases first we characterized each to glider through the tiles, in its form and limits.

Later we initiate fixing a phase, in our case we took the f_{i}_1 phase and we align two tiles T_3 (right illustration of Figure 12). This way the sequence between both tiles T_3 aligned in each one of the four levels is a phase (or sequence) representing a particular structure. Thus we calculate all the periodic sequences in a certain phase.[10]

The procedure determines all the periodic chains to produce a particular structure, the second part is to define what phase we must use, an open problem which perhaps can be resolved using artificial intelligence.

The alignment of tiles is useful to identify four different periodic phases f_i to represent a given glider and each phase determines a different range. But there are gliders with several levels (see *H* glider in Figure 4), they have several phases in which they can initiate from the initial configuration. The second subscript *i* indicates the phase of the T_3 triangle.

The expressions in phases f_{i}_1 are codified of the following way:

$$\#_1(\#_2, f_{i}\_1) \tag{6}$$

where $\#_1$ represents a glider according to the classification of Cook, and $\#_2$ provides the phase of the glider if this it is greater than one. This procedure describes the interaction of any known glider from initial conditions. An important point is that distances between gliders are in mod 4 (number tiles T_3) or mod 14 (number of cells), determined by the period of ether.

3.3 Cycle diagrams

Cycle diagrams are applied in different analysis, an important application is made by Andrew Wuensche and Mike Lesser in the construction of the attraction fields [46]. Howard Gutowitz determines topological properties in these trees in order tomeasure chaos [19].

The calculation made in the cycle diagram is in initial configurations of length smaller than 16, finding several constructions similar to those of the de Bruijn diagram. If $l = 6$ where l it is the length of the initial configuration, we obtain *A* gliders. If $l = 8$ we obtain *B* gliders, but when $l = 16$ we obtain *A* gliders in different phases and groups, with a diagram of 31,336 nodes.

---

[10] The set of regular expressions for all gliders of Rule 110 without extensions and that serves as input file of the OSXLCAU21 system. It is available in [22] or in a text file "phasesGliders R110.txt" from http://delta.cs.cinvestav.mx/˜mcintosh/phpBB2/ (section Rule110)

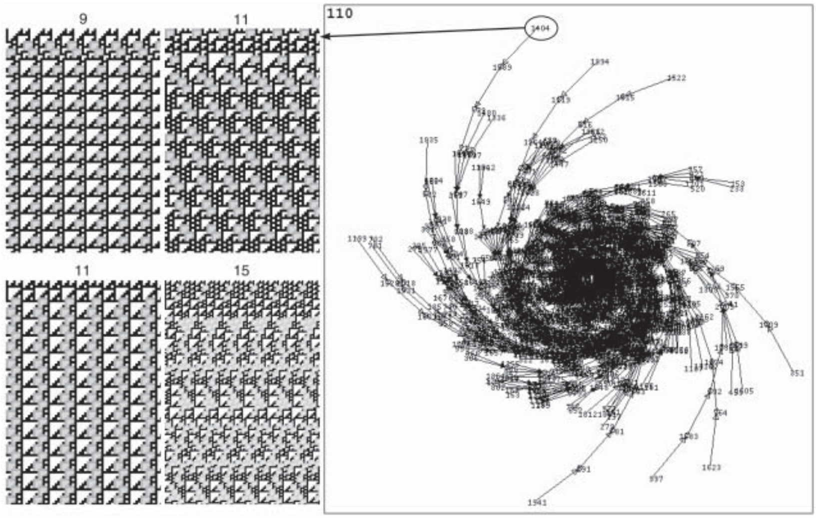

FIGURE 13
Cycle diagram in Rule 110 (note cycle 11).

It is complicated to analyze and plot these diagrams, for example for the de Bruijn diagram we have 1,048,576 nodes in ten generations. Figure 13 illustrates four different examples for the evolutions of the de Bruijn diagram. If $l = 9$,one configuration in the Garden of Eden produces C_1 gliders after 12 generations.

If $l = 11$ we obtain D_1 and C_3 gliders, the difference with the de Bruijn diagrams is that the initial periodic configuration in the cycle diagram produces the glider from its decomposition, meanwhile in the de Bruijn diagram the sequence is directly determined.

If $l = 15$ we have an attractor cycle of 295 nodes and after 39 generations we obtain a periodic background with an internal structure similar to the one *F* glider. But the *F* glider is not constructed from this attractor cycle because its structure is altered, its left margin does not correspond with one *F* glider and the final part is not well-produced due to the very short distance among their parts.

Another way of looking for gliders in Rule 110 is assigning few elements in the initial configuration. Using this idea, it is common to find different periodic regions produced by continuous glider collisions.

For instance, the expression 0*10* creates a glider gun producing groups of A^4 gliders with an *A* glider, Figure 14.

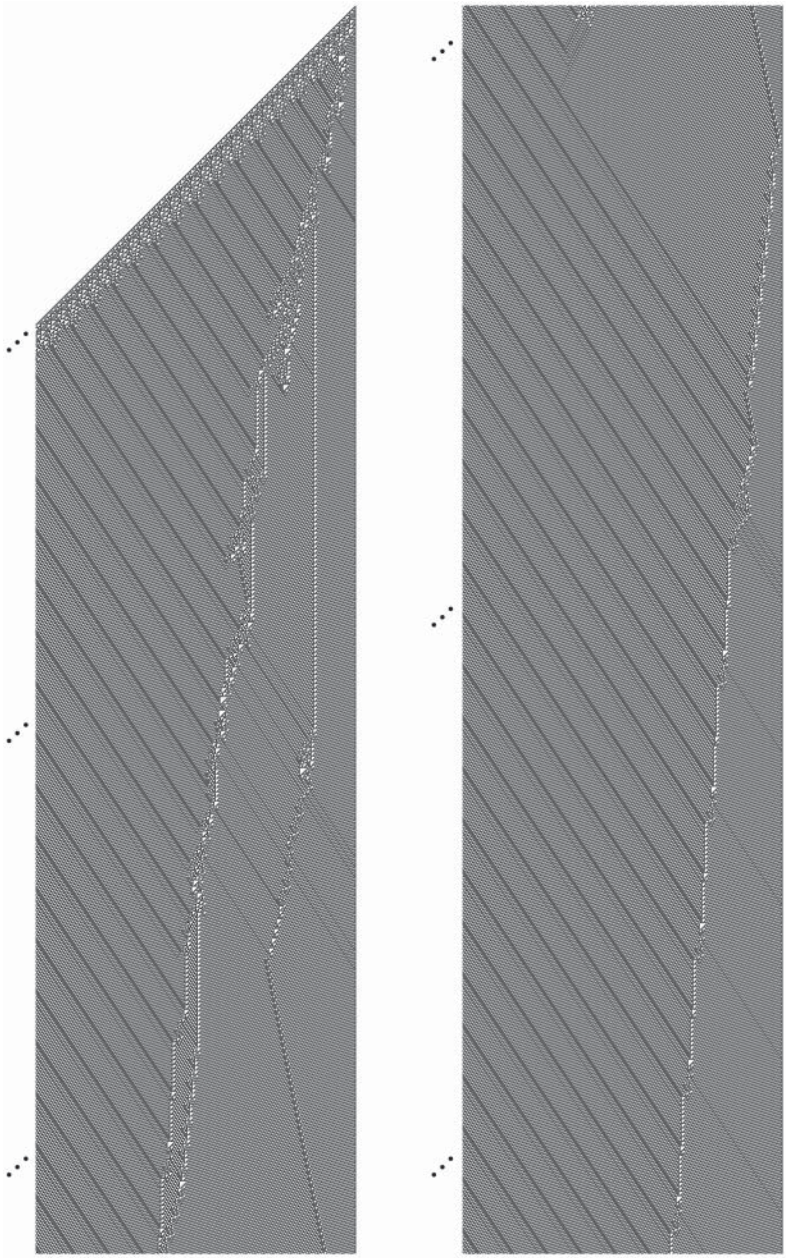

FIGURE 14
Evolution 0*10* in Rule 110.

The evolution becomes stable with the collisions of these gliders in approximately 3,400 generations, producing a periodic region between different glider collisions. The evolution is divided in two parts and cut of the left side because the interesting thing happens in the right part of the evolution. The first evolution shows the first 1,950 generations and the second evolution arrives up to 3,900 generations.

Another alternative is to use the CAM (Cellular Automata Machine [42] of Tommaso Toffoli and Norman Margolus). CAM Is useful to assign different random initial conditions and obtaining quick observations of gliders and their collisions. Few gliders with the same speed show stability in hundreds of generations, emphasizing the E and $\bar{E}$ gliders.

4. PRODUCING COLLISIONS IN RULE 110

In this section we applied the regular expressions determined by the phases for produce and describe collisions in Rule 110. We can find other relevant glider collisions in cellular automata as: The Game of Life [6], Life-3d [4], HighLife [7], Rule 54 [20], Larger than Life [14], and Beehive Rule [49].

We must notice the variety of collisions that we found to produce $\bar{B}^2$, $\hat{B}$, H gliders and the glider gun. A comment from Cook is that $\bar{B}^{n>1}$, $\hat{B}$ gliders and glider gun cannot be produced through collisions. In [24] we have at least one production for each glider. Nevertheless, it is possible to obtain more than one production for these complicated structures, including an extensible glider gun.

4.1 Producing gliders

In order to find the production of a particular glider we begin reviewing the list of all the binary collisions [23]. In the case of $\hat{B}$, H gliders and the glider gun, specialized searches were developed.

A glider

Figure 15 illustrates two ways of yielding an A glider, the first is between a C_1 glider against one H glider, here a complicated structure as the H glider can almost be contained by a relatively simple collision.

The second case is between D_2 and F glider. The A glider appears very frequently in the evolution space, but to produce it in an isolated way

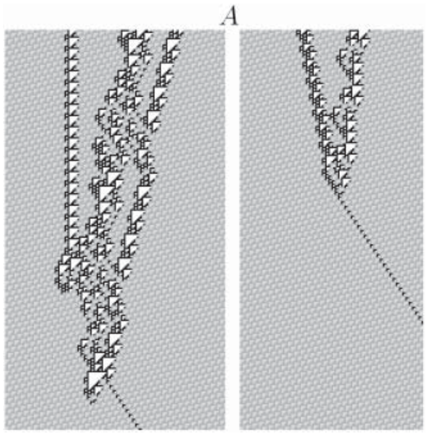

FIGURE 15
Producing A glider.

is only possible in two cases by a binary collision. The A glider has the maximum positive speed of 2/3.

B, $\bar{B}$ and $\hat{B}$ gliders

Figure 16 illustrates two ways of producing a B glider. The first is between a D_1 against one H glider. The intervention of the H glider is only necessary to produce the A, B and C_2 gliders.

The second production is between one F against one $\bar{E}$ glider; most of the collisions between these two gliders form a soliton [23].

The third figure illustrates the production of a $\bar{B}$ glider among a group of A^2 gliders against a G glider. This is one of gliders that cannot be obtained through a binary collision, its structure is a little bit complicated since all the parts of the $\bar{B}$ gliders move with speed −1/2 which is the maximum negative speed.

The fourth figure depicts the production of a $\hat{B}$ glider, this glider was the last found as a product of some collision. The composition of this collision is both complicated and rare. It begins with one A colliding one F, the closeness of the $\bar{E}$ glider arriving from the right creates an almost periodic region in the struggle of two $\bar{E}$ gliders to become stable. Nevertheless, this chaotic region allows creating the needed elements to interact in a triple collision with D_1 and C_2 for producing a $\hat{B}$. In this production there are superfluous A and D_1 gliders which survive in the collision, these gliders are erased by $\bar{E}$ and B gliders.

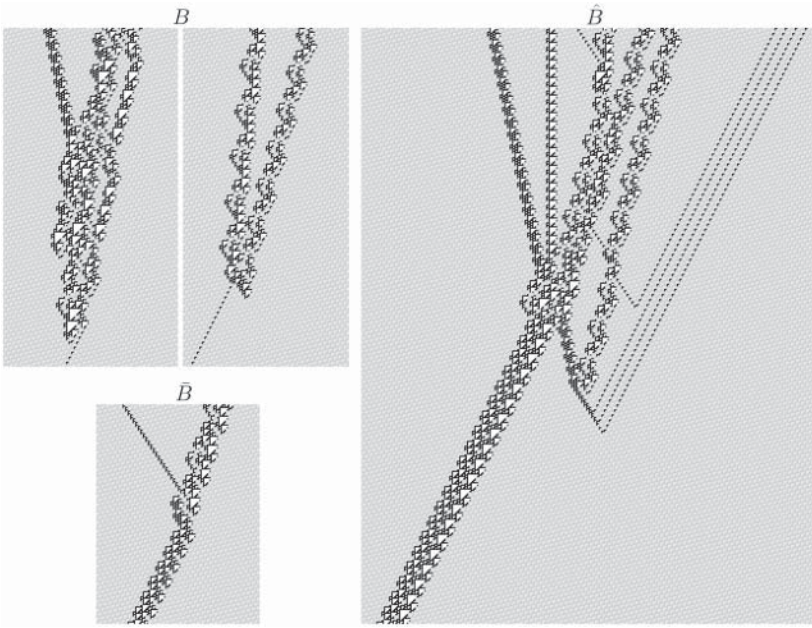

FIGURE 16
Producing B, $\bar{B}$ and $\hat{B}$ gliders.

The collision is really complicated and we have the problem of different speeds in the beginning of the production with the A and C_3 gliders, implying a non-proper collision [40]. Other different and complicated example of reproducing a $\hat{B}$ glider can be consulted in [24].

C_1, C_2 and C_3 gliders

Figure 17 illustrates the production of C gliders, these gliders are the unique ones with no shift. C_1 glider can be yielded in two ways, the first case is the collision between one A glider against a C_2 glider. The second case is between one F glider against a $\bar{B}$ glider, although firstly a C_2 glider is produced, later on one A glider arrives transforming it into a C_1 glider just as the previous case.

C_2 glider can be obtained from six ways, the first case is between one A against a $\hat{B}$ glider. We know that the collision between one A glider against a B cancels both gliders, also in one of its three collisions the A can be cancelled with one $\bar{B}$; something that does not happen with the $\hat{B}$. The third case is between one A against a D_1, observe that the last

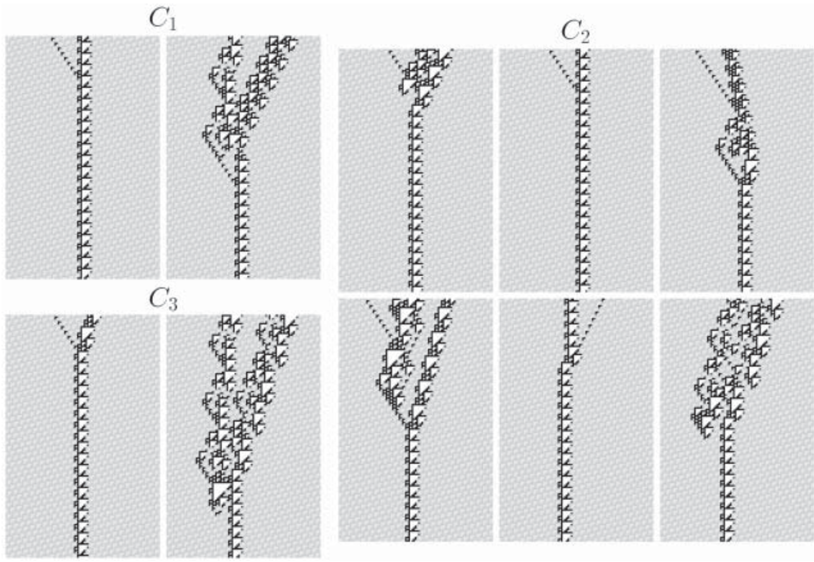

FIGURE 17
Producing C_1, C_2 and C_3 gliders.

collision fragment before forming the C_2 is between one E and a group of A^2 gliders. The fourth exampleis between one A against an H, again we can see in the final part a collision between E and A^2 gliders. The fifth case is between a C_1 against a B increasing the index of C. The sixth case is between $\bar{E}$ and G gliders.

C_3 glider can be produced in two ways, the first case is a collision between one A against one E and the second case is between F and G.

D_1 and D_2 gliders

Figure 18 illustrates the productions of D_1 and D_2 gliders. The first case forming D_1 is between one A against a D_2, this case decrements the subindex of D. The second case is between one A against an E. The thirdcase is between a C_2 against a B and the fourth case is between one F against a G. Although some of them look like simple collisions, altogether can construct long chaotic regions.

D_2 is another glider which cannot be produced from a binary collision, nevertheless a careful search in the list of binary collisions helped to find it out. It is formed by a triple collision between one F against a G and a B turning the G intoa G^2 glider. G glider is extendible as the E glider with the

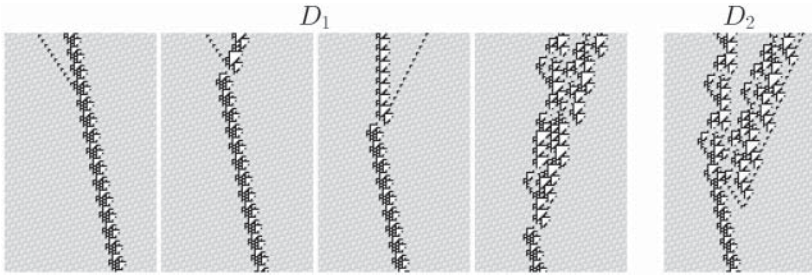

FIGURE 18
Producing D_1 and D_2 gliders.

T_5 triangles in its right margin. This extension carries B gliders for each T_5 that it finds, finally the B cancels the exceeding A to isolate the D_2 glider.

E and $\bar{E}$ gliders

Figure 19 illustrates the production of E gliders. The first case is between a C_3 against a B, the second case is between a D_1 against a B. We can construct periodic series by means of collisions, for example: $D_1 \leftrightarrow B = E$, $A \leftrightarrow E = C_3$, $A \rightarrow C_3 = C_2$, $A \rightarrow C_2 = C_1$, $C_1 \leftarrow H = A$ and $A \leftrightarrow D_2 = D_1$. Taking a large part of the list of gliders, it is possible to continue producing an unlimited series.

The third case is between a D_1 against a $\bar{B}$, notice that the T_8 triangle in the E glider is the same that the one of the $\hat{B}$ glider. Tile T_5 appears in both $\bar{B}$ and $\hat{B}$ gliders, a good adjustment of this tile produces $\bar{B}$ and $\hat{B}$ extended gliders. These extensions are difficult to find in the evolution space.

$\bar{E}$ glider can be obtained from three ways, the first case is between one A against one F. Observe that a T_{10} tile is generated in the process; one question is if it is possible to describe some procedure using distributed

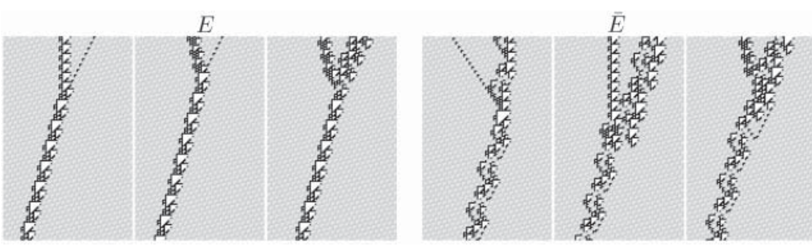

FIGURE 19
Producing E and $\bar{E}$ gliders.

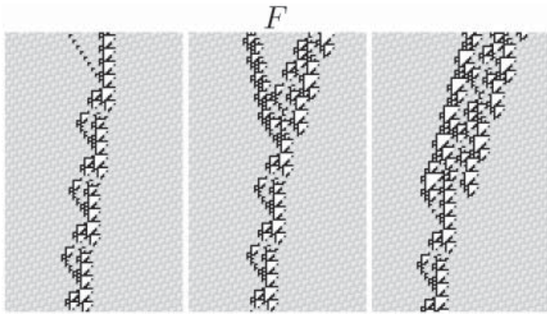

FIGURE 20
Producing F glider.

large triangles on the evolution space [34]. The second case is between a C_3 against a G, the third case is between a D_1 against a G. Reviewing the other collisions between both gliders, it is possible to generate the T_{10} tile producing one $\bar{E}$ as in the first case. Finally the existence of an isolated T_{10} tile implies one $\bar{E}$.

***F* glider**

F glider has three ways to be generated as Figure 20 illustrates. The first case is between one A against a C_1. Experimental results suggest that this collision can be considered as a base to form the F glider, because all the productions generating F reach the basic collision A against a C.

The second case is between a D_2 against a G, in the case of one $\bar{E}$ glider (previous figure) the production is between a D_1 and a G, these gliders are in the same phase, the only difference is the index of D and therefore a different result, this is an example where the change of a single value changes the whole production. The third case is between one E against a G, in the development of the collision we have the first case between one A against a C_1. In fact, the second and the third case have the same collision that the first one.

***G* glider**

G glider can be produced between a D_2 against one E as Figure 21 shows. In this production we have a T_{13} tile and like in the $\bar{E}$ glider, an isolated T_{13} implies a G glider. We illustrate other examples which although are not binary productions, represent different forms to produce a G.

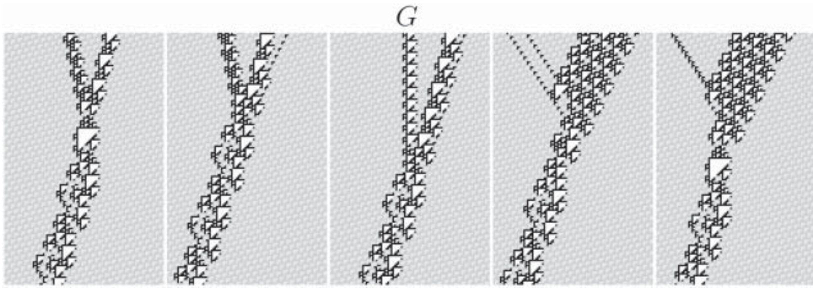

FIGURE 21
Producing G glider.

The second case is between a D_2 against one E^2, looking for periodic sequences we can analyze the following reactions: $D_2 \leftrightarrow G = F$, $D_2 \leftrightarrow F = A$, $A \leftrightarrow C_1 = F$. The third case is with a C_3 against one E^2, in this production the beginning of the collision is different from the second case, nevertheless later the central part is the same one. The fourth case is between two A gliders and a $\bar{B}^3$, the fifth case is between a group of A^2 gliders and a $\bar{B}^3$. These gliders are the same but they are in different phases and distances, however producing the same final result.

H glider

Figure 22 illustrates eight cases to produce one H, in the first case the H is calculated from natural gliders, initiating from a group A^2 colliding with a C_1 producing an isolated T_{10} tile which produces one $\bar{E}$. Nevertheless it is affected by a B and later by another A forming the left margin of the H glider; finally a group B^4 arrives constructing the right margin.

Based on this example we may suppose the possibility of constructing one H with an $\bar{E}$, an A and B gliders. This production is possible although with small changes and can be consulted in [24].

The second case is also produced with natural gliders, similar to the first case. In this case a D_1 collides against a group of B^3, the difference with the previous case is that a T_8 tile is generated, then one A and a group of B^4 gliders arrive and the production is equal to the previous one. The third case seems to be the first intuition to produce one H because its right margin is formed by an E which is destroyed and reconstructed periodically. The collision is between one F and two E gliders and we have a direct production of H.

FIGURE 22
Producing H glider.

The fourth case initiates with one $\bar{E}$ and two D gliders producing a chaotic region prolonged by two A gliders producing a T_{13} tile in the process. Another chaotic region is produced but immediately controlled by a group of B^4 gliders generating at last the H. The fifth case begins with one F and B gliders, the collision must produce a $\bar{B}$ returning to the F, but the group of A^2 and B^4 gliders arrives to produce the H.

The sixth case initiates with the collision between a C_3 and one F but simultaneously two B gliders arrive from the right and a D_1 arrives from the left producing one H without additional structures. The seventh case begins with one F and two B gliders, the produced region is immediately affected by $\overline{B}$ and A gliders. The collision yields one A which does not take part in the process anymore, therefore we add a B to eliminate it.

The last case is between one F receiving collisions on both sides at the same time by two D_1 and two B gliders. The initial part of the collision before producing the H is similar to the fourth case, nevertheless is not the same because they have different ways to arrive at the same part.

Glider gun

In The Game of Life, a glider gun is useful to simulate logic operations in the evolution space; thirty gliders of five cells are carefully distributed in the evolution space to construct the glider gun [6].

In [24] we obtain a glider gun through a collision, this one only with natural gliders. Cook asserts that the glider gun in Rule 110 does not have some immediate utility. Figure 23 depicts six examples to construct the glider gun.

The first case is through a triple collision among D_1, C_1 and $\overline{E}$, this example is the cleanest case and the glider gun is directly produced. The second case initiates with one F colliding one A, this collision must produce one $\overline{E}$ originated by the T_{10} tile affected later by the group of B^4 and A^5 gliders. The collision seems familiar with the first example in the H glider, but the difference is that these collisions are produced in different phases and with a distinct number of gliders. The third case is between two merged C_2 and C_3 gliders colliding against one F affected before by three B gliders. The chaotic region is controlled by a group of B^4 gliders producing a glider gun, the process yields one B cancelled by one A. The fourth case is between one $\overline{E}$ and a group of B^4 gliders, the produced region is controlled by a group of A^5 gliders yielding a glider gun.

The fifth case is almost identical, the difference is the group of B^4 gliders colliding in another phase with the $\overline{E}$ glider. The phenomenon is originated because T_5 tiles only carry B gliders without affecting the process. The sixth case is a little more complicated in construction, it initiates with one $\overline{E}$ colliding two B gliders producing a T_{10} tile which is quickly affected by a group of B^3 gliders and one A arriving from the left. One can identify two D gliders in the central part of the collision, but their existence is very short because they take part in another collision generating both a T_9 tile and one F. Later a $\overline{B}$ collides in the inferior part of the tile and produces two B gliders colliding against the F, this results in a D_1 colliding against the chaotic region and finally producing the glider gun. The A produced in the process is cancelled by a B.

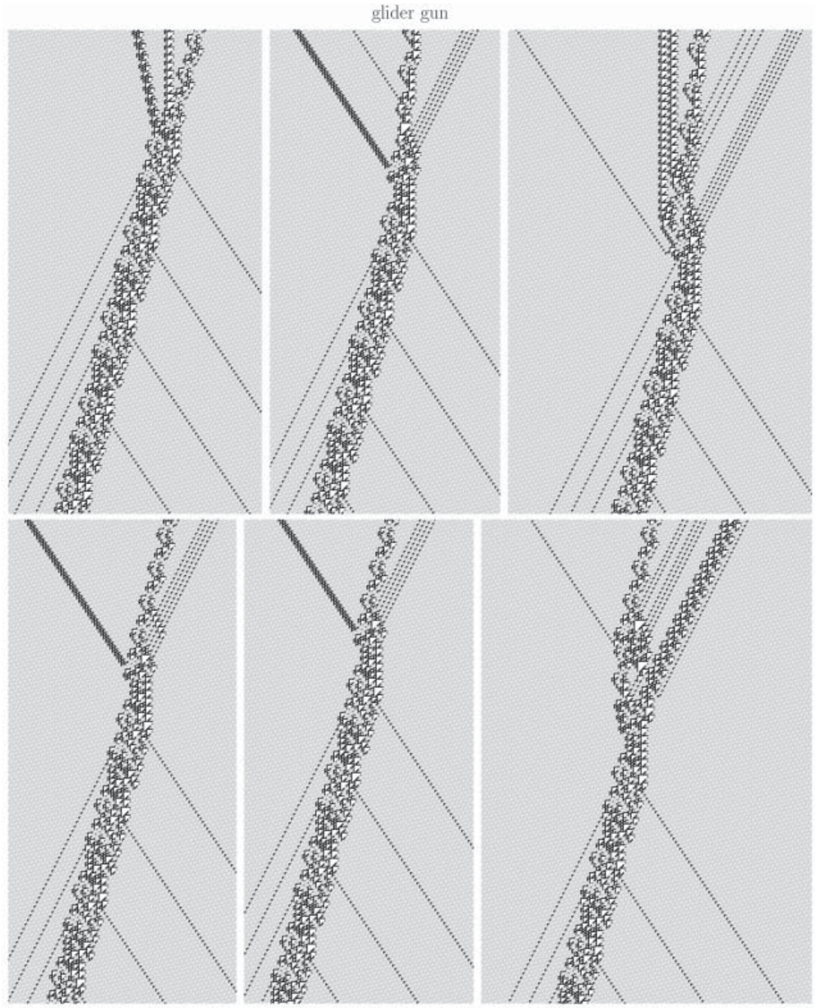

FIGURE 23
Producing glider gun.

In the productions of the glider gun we have the same initial part in all cases, in particular the column of T_6 tiles before the T_8 tile; this is part of the right margin in the glider gun. Analogously the column of T_4 tiles in almost all the cases in its left margin is useful to determine the central part of the glider gun.

Glider gun″

This is an example discovered on searching for a collision to produce a glider gun, a phenomenon which can be seen as a new extendible glider. The example illustrated in the following figure is complicated and several details must be taken into account. The problem is to demonstrate that in any case the $\bar{E}$ glider can always be conserved on the right side of the glider gun.

A first test[11] is assigning several $\bar{E}$ gliders in different phases and verify the previous idea, because although the $\bar{E}$ glider is faster than the glider gun, the A gliders periodically generated by the glider gun change the position of the $\bar{E}$ in steps of two cells keeping the distance between both gliders.

Other question is to find a set of productions with the glider gun originating one $\bar{E}$ from natural gliders. Reviewing the binary productions we can obtain it as Figure 24 describes.

The production begins with a D_2 colliding the first A and it becomes a D_1, later the second A transforms it into a C_2. The third A changes it into C_1, the fourth A transforms it into F and finally the fifth A yields the $\bar{E}$ which must always travel on the right side of the glider gun. The first $\bar{E}$ arriving from the right is introduced with a suitable distance, the intention is to place this glider in an adequate phase for illustrating the two ways in which the A can cross the $\bar{E}$ as a soliton.

A question raised by Fred Lunnon is to know if $\bar{E}$ in all its possible phases is able to produce the same phenomenon. The answer is no; there exist two cases in which the A glider does not across $\bar{E}$ as a soliton. In these cases an E^2 is produced. What is important is that the glider gun always implies an $\bar{E}$, even from an initial condition where the $\bar{E}$ is not in an adequate phase but the A gliders are useful to adjust it. Figure 24 show some examples of captures of this sort.

This production loses two times the phase of the $\bar{E}$, first it is transformed into the following sequence of gliders by each collision with the A gliders: E^2, E, D_1, C_2, C_1, F and finally returning into $\bar{E}$. From D_1 we have the same succession of productions that in first case, but the $\bar{E}$ glider does not get the adequate phase yet. Then although two A gliders cross it as solitons, the third one transforms the glider into E^2 and the same series of productions yield the final $\bar{E}$ glider.

---

[11] Initial condition: e*-gun(C2,f_1_1)-D_2(A,f_4_1)-13e(f_1_1)-$\bar{E}$(B,f_1_1)-e-$\bar{E}$(A,f_1_1)-e*

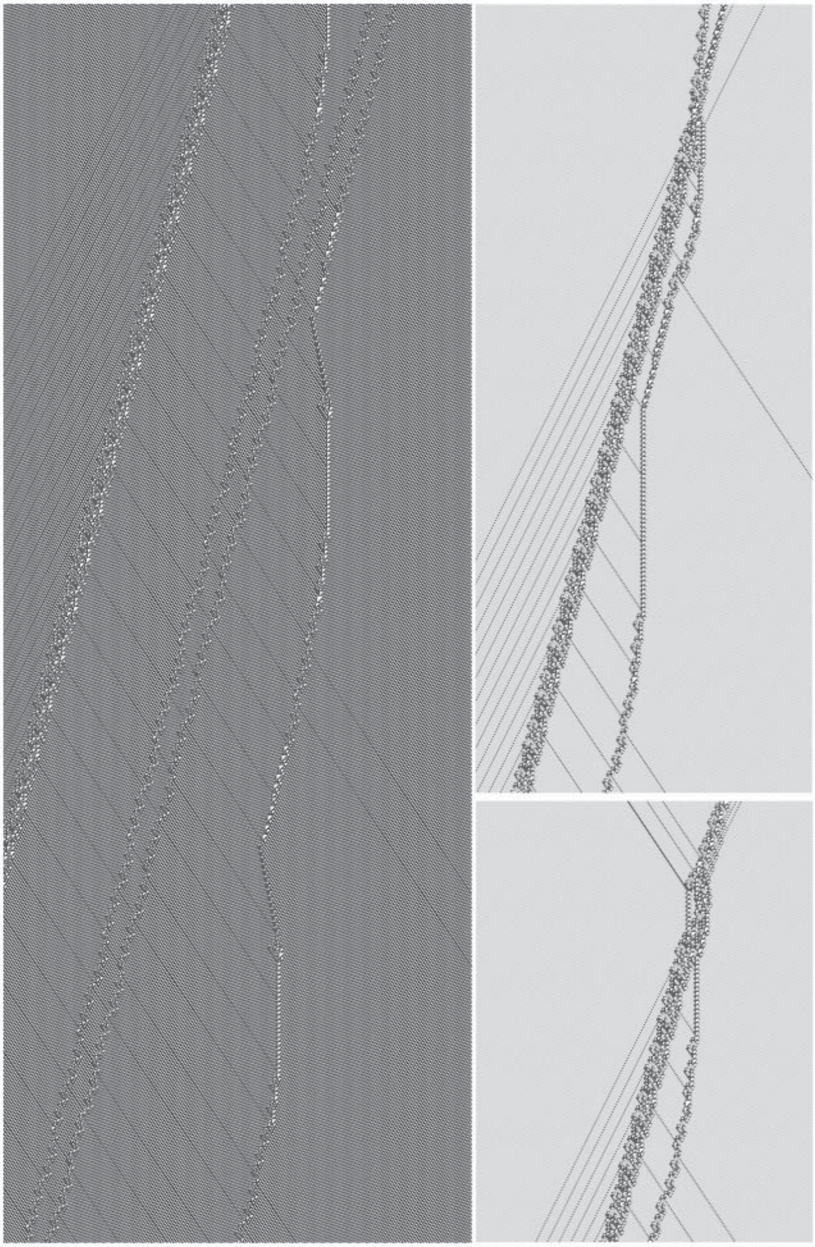

FIGURE 24
Producing glider gun^{n}.

The second panel in Figure 24 illustrates that the glider gun$^{n=1}$ can be constructed by a collision. The production is not immediate and needs several generations. It initiates with one F colliding against one E^2, the chaotic region so produced is very sensitive. When the B arrives on the right, the glider gun$^{n=1}$ is yielded in the contact point, if the B glider is not present or arrives in another phase, the production is totally different.

The third case is faster although it is not less complicated, the collision is between a G^4 against two spaced A and a group of A^2 gliders. Finally the collision between the C_3 and the chaotic region yields the glider gun$^{n=1}$.

Every glider in Rule 110 takes part at least in one collision for producing another glider, from A to H. We consider independently the glider gun because its interactions with others gliders are complicated by the A and B gliders periodically produced.

Table A.1 (appendix) shows all the glider sequences up to now known to form each glider in Rule 110. With these relations we can look for a path from one glider to another crossing all the list. This is possible although the order is not strictlyconserved and we may have several combinations.

4.2 Groups of gliders by collisions

The problem of covering the evolution space with triangles is related as well with the structure of gliders and periodic regions in the de Bruijn diagrams (Figure 8).

Figure 25 presents some productions to obtain pairs or groups of gliders. The first case is among a group of A^5 gliders colliding against one $\bar{E}$, yielding a chaotic region controlled by a second $\bar{E}$ producing two merged $\bar{B}$ gliders. The second case between one A against a $\hat{B}$ producing three merged C gliders, two C_3 and a C_1. The third case is among two spaced A gliders and a G, this collision produces two merged D_1 gliders.

The fourth case is between one A against a $\bar{B}$ producing two $\bar{E}$ gliders. The fifth case yields several E gliders, the collision begins with a C_3 against one F and two spaced B gliders. The central part generated by the chaotic region is transformed by a D_1 into $\bar{E}$ and E^2 gliders, on the other side the second B against a D_1 transforms it into one E.

The sixth case is between a D_1 against one E^3 producing two merged E gliders. The seventh case is among two D_1 against a $\bar{B}$ producing a pair of F gliders, we can also identify the structure of two $\bar{E}$ gliders after the collision, however they are conserved. A fact is that a pair of two $\bar{E}$ gliders is transformed into a pair of F gliders by a collision.

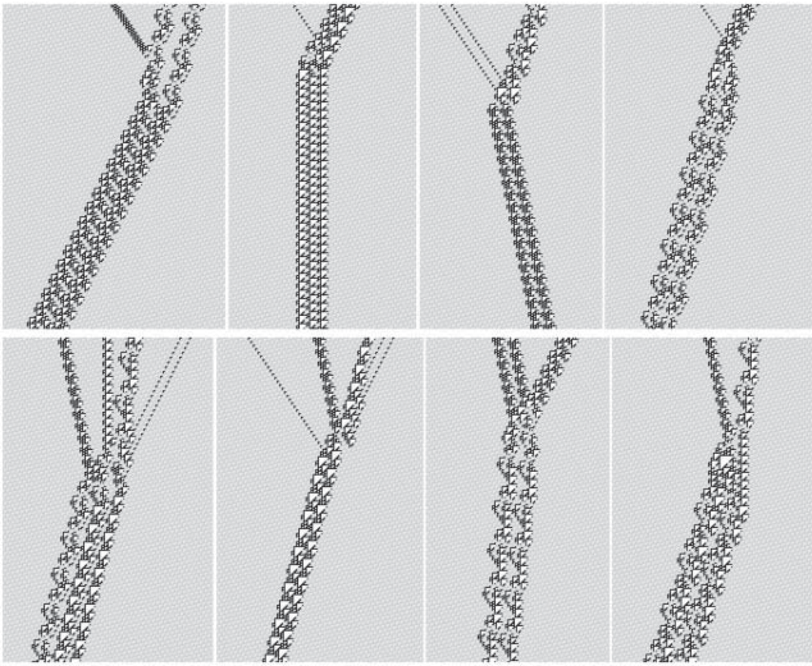

FIGURE 25
Constructing groups or packages of gliders.

The last case is between a D_2 against one F, it yields two G. In this collision we can see a C_2 comprising the F and it continuous because there are not A gliders from the left for returning it into F.

4.3 Gliders with extensions

There are gliders in Rule 110 with extensions, at first instance this implies an unlimited number of collisions among gliders in Rule 110, without forgetting that we can take groups of gliders. Some of these extensions are easily obtained but some others are really difficult to construct. Gliders with these features are $\bar{B}^n$, $\hat{B}^n$, E^n and G^n. Extensions for E^n and G^n are frequently generated in the evolution space as Figure 2 illustrates.

The way of decrementing and increasing one E^n is not complicated, E^n is transformed into E^{n+1} by a collision against a B; to obtain E^{n-1} is enough a collision against one A. This set of collisions may be interpreted as a binary counter, the problem is that we do not have a periodic region for the operators and another for data. G glider can be increased into G^{n+1} colliding against a B, but the opposite case is not obtained by a collision with one A as the case for the E glider. Just when $n = 2$, G can be decremented into G^{n-1} colliding with one A; in any other case G is destroyed.

The extensions for the $\bar{B}^n$ and $\hat{B}^n$ gliders are rare and more complicated to find in the evolution space as n grows, both for $\bar{B}^n$ when $n \geq 2$ and for $\hat{B}^n$ when $n \geq 1$. A computational search demonstrates that $\bar{B}^2$ can be produced by means of collisions as Figure 26 shows. The first case is among D_1, C_1 and $\hat{E}$, the process forms A gliders which are cancelled by four B gliders. The collision is very similar to the one yielding a glider gun, the difference is that the phases are distinct and the first B arriving from the right is the one determining the $\bar{B}^2$ glider.

The second case is more complicated, it needs a pair of F gliders in the center, a C_3 on the left useful to form the left margin of the production with two A and a group of B^4 gliders arriving from the right with a $\bar{B}$. The chaotic region produces eight A gliders cancelled in order to isolate the

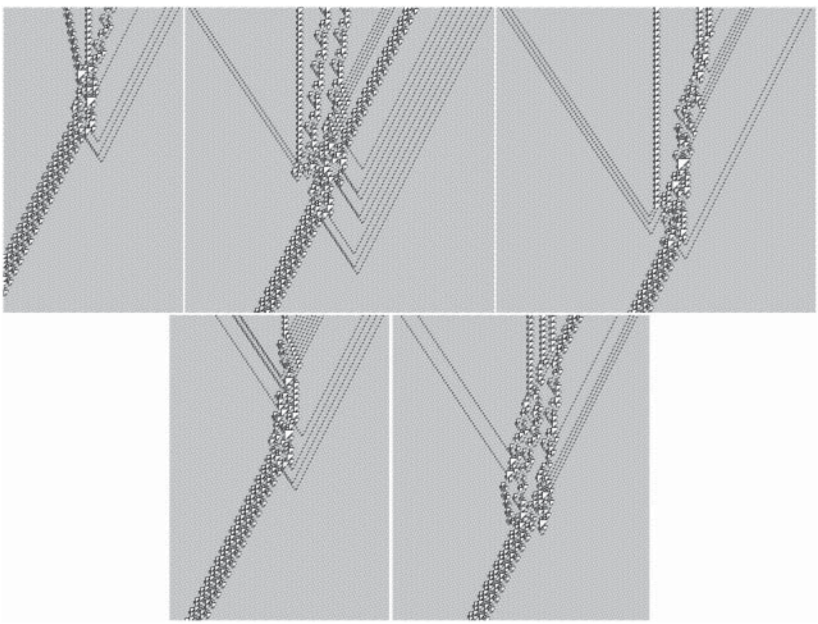

FIGURE 26
Producing $\bar{B}^2$ glider.

$\bar{B}^2$. In the third case, the central part is determined by a decomposition raised by the collision among two C_1 against a B. Then the group of B^4 gliders cancels one A affecting as well the chaotic region producing a T_{13} tile. The third C_1 glider on the left determines the production of the $\bar{B}^2$.

The fourth case is produced by natural gliders, the collision begins with a group of $8B$ gliders colliding against a C_2, then two groups of A^2 gliders and one A determine the chaotic decomposition yielding a $\bar{B}^2$. Perhaps this collision can be reduced initiating with a D_1 and the group of $8B$ gliders, because the T_5 tiles do not take part in the process and they just carry the B gliders.

The last example is another complicated construction, the central part is composed by known gliders. The production initiates with a $\bar{B}$ against a C_1 producing two F gliders and a B colliding the C_1 transformed by another B. The chaotic region so produced goes against two C_2 gliders yielding two $\bar{E}$ gliders, the first collides against two A gliders transforming the second $\bar{E}$ into one F. The D_1 and F determine the upper part of $\bar{B}^2$, but the inferior part is defined by the decomposition of the F with four B gliders. This is a clean production because it does not produce additional gliders in the process.

The $\bar{B}^3$ has been found in several constructions, nevertheless these were not the product of a given collision. The $\bar{B}^4$ can be constructed from a very short decomposition; in both cases these gliders cannot be produced of isolated way [21], for $n \geq 5$ we have not found any construction. In the case of the $\hat{B}^n$ for $n \geq 3$ we have not examples yet.

Figure 27 shows the collision to obtain a $\hat{B}^2$ but not of isolated way because there are exceeding C_1 and F gliders. The collision is complicated and requires several intermediate processes to calculate the final gliders. This is an example of several synchronized collisions arriving in the right moment and with suitable phase. The first C_1 controls the decomposition generated by the collision among the pair C_2 and $\bar{E}$, this one also produces a group of A^2 and A gliders colliding a pair of C_2 and one $\bar{E}$ glider generated by the collision of A, F and $\bar{E}$ gliders on the right. A pair of chaotic regions yield the $\hat{B}^2$. In this way the nature of these extensions is well-represented by collisions.

An open question is whether the $\bar{B}^n$ and $\hat{B}^n$ gliders have some way to increase and decrement their index as it happens with the E^n glider.

We don't have a way for increasing the index of $\bar{B}^n$ and $\hat{B}^n$. Nevertheless we can decrement it for both gliders by a periodic collision with a D_2 as Figure 28 depicts. The expression codified in phases is for both gliders (Table 3 - A.3).

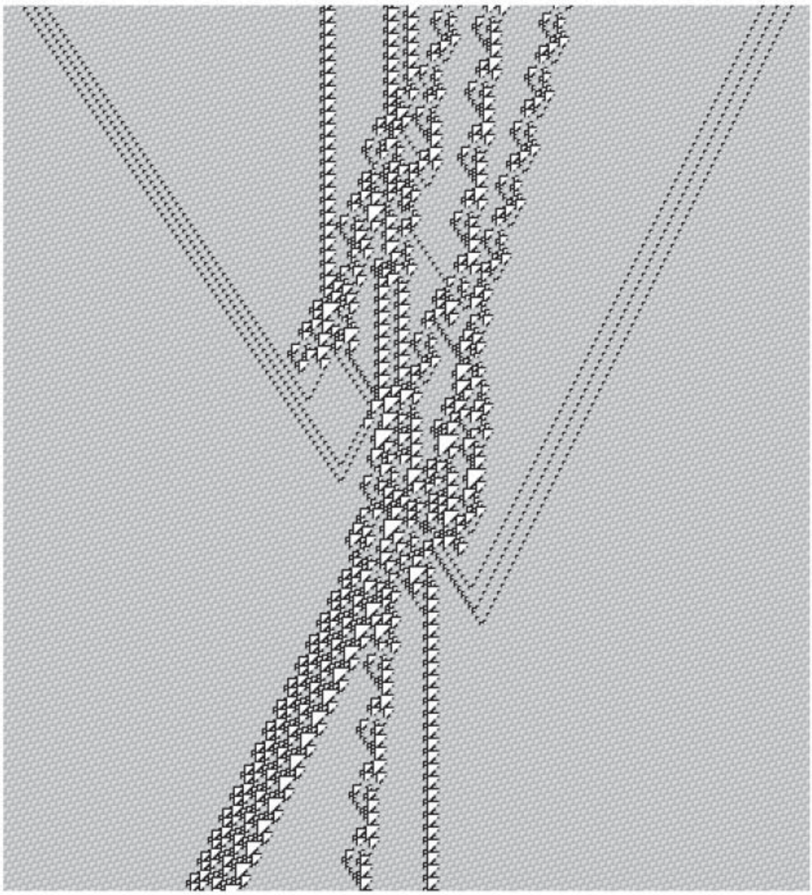

FIGURE 27
Producing $\hat{B}^2$ glider.

The extension in $\overline{B}^n$ and $\hat{B}^n$ is made in the central part of the glider, maintaining fixed the upper and the lower part. Thus the periodic margin is fixed colliding with a D_2 glider, eliminating the upper part and taking the following central part into a new upper part.

5. CONCLUSIONS

The existence of new gliders in Rule 110 is still an open problem. Then all gliders until now known can be produced by one or several collisions

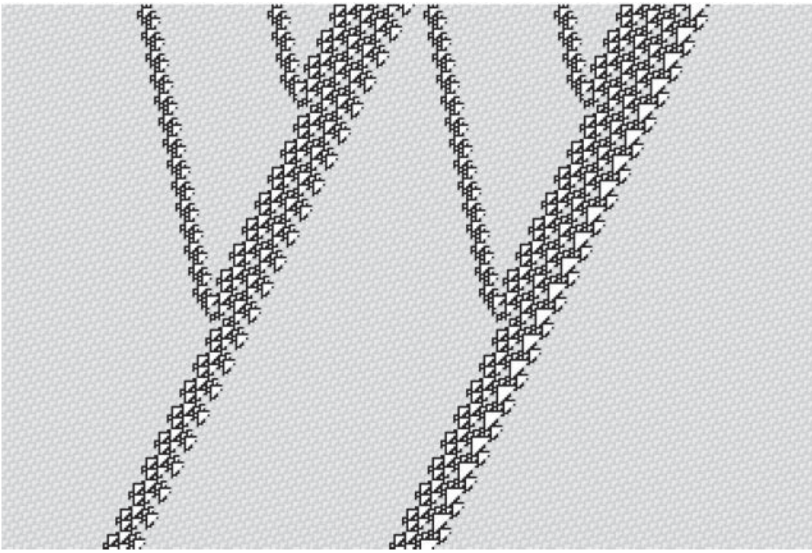

FIGURE 28
$\bar{B}^{n-1}$ and $\hat{B}^{n-1}$ gliders.

and some gliders with complicated extensions. The productions presented in this paper are codified from initial conditions through periodic sequences.

On the other hand, universality in the strict sense can sometimes be demonstrated, although establishing an adequate system of collisions for Rule 110 shows how arduous the process can be [25]. The possibility, ranking as an alternative route to Church's thesis [9]. Universal computation in cellular automata was developed by John von Neumann in his model of twenty nine states [44]. In the one-dimensional case we have several important contributions in [29] and [37].

Some questions on the universality in Rule 110 can be discussed. For example, does Cook have one of Kudlek-Rogozhin universal circular Post machine? [27]. Cyclic tag system is a variant of tag systems, nevertheless we have not found direct antecedents in this direction. The universality of tag systems is demonstrated with two element state set by John Cocke and Marvin Minsky in [13] and [36]. The construction of Cook seems new, it is interesting that the pair of productions in this system is undecidable, an analogous result to Emil L. Post [39] and which must be discussed elsewhere.

Partial results making computations with the cyclic tag system as the Fibonnacci sequence have been realized by Paul Chapman (January 2003). However we must develop results as parenthesis balancing, constructing binary counters and discussing their implementations in Rule 110. Fred Lunnon has implemented a system to produce collisions between *E* and *H* gliders in Rule 110 (February 2003). Mirko Rahn discusses in [41] a formal way the operation of the cyclic tag system with regard of the Turing machine [43], although this approach is oriented to implement a function accepting some Turing machine for producing an initial configuration in Rule 110 simulating the operation of this machine.

Besides, we have a configuration producing a copy of one initial glider through collisions. We can found some relations between the ants of Cristopher G. Langton [28] and gliders in Rule 110 as show Figure 29.

We have an initial structure represented by a *G*, from three collisions with others gliders we conserved the original one and we obtain as well a new *G* glider. In addition the process can be unlimited produced in a constant way.

It begins with a *G* receiving a *B* transforming it into a G^2, then a $\bar{B}$ collides against the G^2 producing a *G* and one *A*. Two *B* gliders are added to cancel the *A* and produce a G^2 with the new *G*. The expression for the evolution is: e*-G(A,f_1_1)-B(f_1_1)-e-{$\bar{B}$(B,f_1_1)-2B(f_1_1)-e-$\bar{B}$(A,f_1_1)-2B(f_1_1)-e-$\bar{B}$(C,f_1_1)-2B (f_1_1)-e}*.

A further work is to project Rule 110 in more dimensions, this can be useful in order to find some properties which may not be clear in the one-dimensional case, in analogy to the projection of Life in three dimensions made by Carter Bays in [4,5]. In the three-dimensional case we can expect an evolution space covered by tetrahedra or some other geometric form forming ether and the set of gliders, a tiling problem.

Rule 110 is not well-understood yet because in spite of being the automaton of lowest order supporting universal computation, its complexity is not shared by any other cellular automaton of order (2,1). Rule 110 has its own complicated collision theory due to the infinite number of possibilities.

In some aspects, it is difficult to apply the traditional analysis in cellular automata for Rule 110. It is necessary to have previous experience analyzing the rule. Comparisons with Life are found in a game of words. Nevertheless Rule 110 displays its own problems of representation,

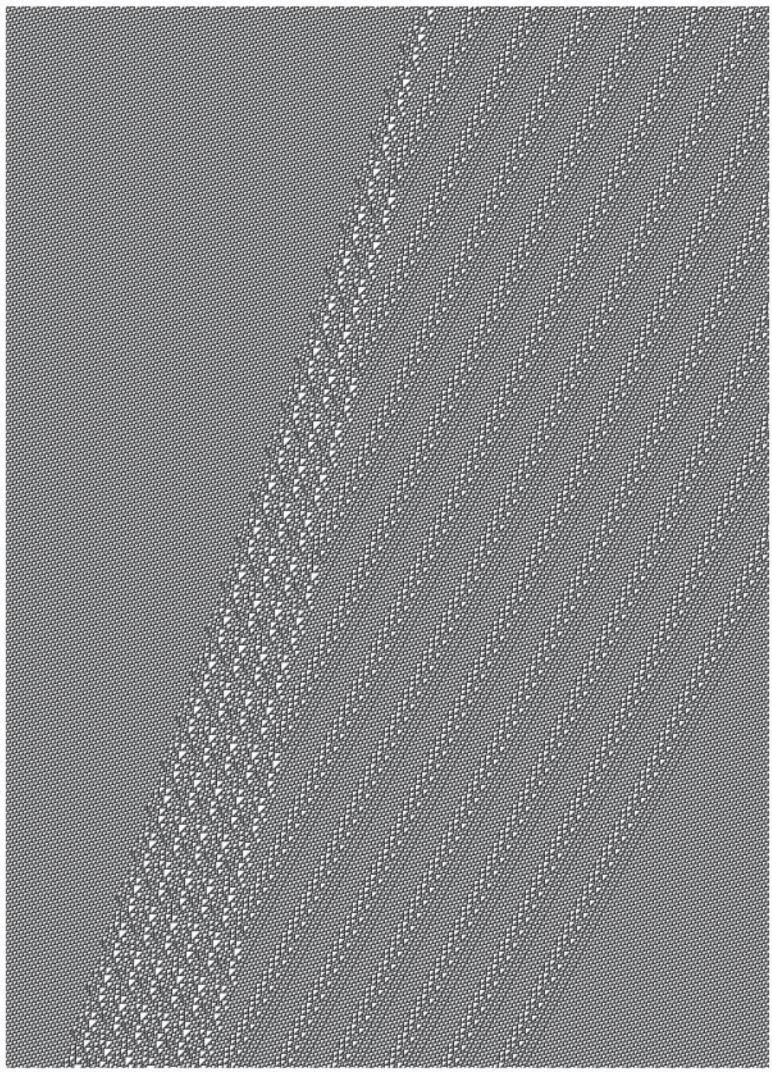

FIGURE 29
Another extension in Rule 110.

constructions and complexity. Thus Rule 110 has its own space properly not to be compared like a rule of the Life type.

Finally the production of meta-gliders, solitons, large triangles and several Rule 110 objects by collisions will be displayed in a next article.

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A. Appendix

A.1 Table 1

Relation of productions in phases for each glider of Rule 110. The first column indicates glider and second column the code in phases to reproduce the collision.

A

C_1(B,f_1_1)-e-H(A,f_1_1)
D_2(A,f_1_1)-e-F(A,f_1_1)

B

D_1(C,f_3_1)-2e-H(B3,f_1_1)
F(G,f_1_1)-e-$\bar{E}$(C,f_1_1)

$\bar{B}$

A^2(f_1_1)-3e-G(A2,f_1_1)

$\hat{B}$

C_2(B,f_1_1)-D_1(A,f_1_1)-e-$\bar{E}$(B,f_1_1)-4e-$\bar{B}$(C,f_2_1)-3B(f_1_1)
D_1(C,f_1_1)-2e-C_2(B,f_3_1)-A(f_1_1)-e-F(C,f_1_1)-$\bar{E}$(F,f_1_1)-$\bar{E}$(D,f_1_1)-5e-4B(f_1_1)

C_1

A(f_1_1)-e-C_2(A,f_1_1)
F(A2,f_1_1)-e-$\bar{B}$(A,f_1_1)

C_2

A(f_1_1)-e-$\hat{B}$(B,f_1_1)
A(f_1_1)-e-C_3(A,f_1_1)
A(f_1_1)-e-D_1(A,f_1_1)
A(f_1_1)-e-H(A,f_1_1)
C_1(A,f_1_1)-e-B(f_1_1)
$\bar{E}$(F,f_1_1)-G(A,f_1_1)

C_3

A(f_1_1)-e-E(B,f_1_1)
F(G,f_1_1)-e-G(E,f_1_1)

D_1

A(f_1_1)-e-D_2(A,f_1_1)
A(f_1_1)-e-E(A,f_1_1)

C_2(A,f_1_1)-e-B(f_1_1)
F(G,f_1_1)-e-G(A,f_1_1)

D_2

F(G,f_1_1)-e-G^2(B,f_1_1)

E

C_3(A,f_1_1)-e-B(f_1_1)
D_1(A,f_1_1)-e-B(f_1_1)
D_1(C,f_1_1)-e-$\bar{B}$(A,f_1_1)

$\bar{E}$

A(f_1_1)-2e-F(E,f_1_1)
C_3(A,f_1_1)-e-G(E,f_1_1)
D_1(C,f_1_1)-e-G(A,f_1_1)

F

A(f_1_1)-e-C_1(A,f_1_1)
D_2(C,f_1_1)-e-G(A,f_1_1)
E(A,f_1_1)-e-G(B,f_1_1)

G

D_2(A,f_1_1)-e-E(D,f_1_1)
D_1(A,f_1_1)-e-E^2(D,f_1_1)
C_3(A,f_1_1)-e-E^2(B,f_1_1)
A(f_3_1)-A(f_1_1)-2e-$\bar{B}^3$(B,f_1_1)
A^2(f_1_1)-2e-$\bar{B}^3$(A,f_1_1)

H

A(f_1_1)-7e-A(f_3_1)-3e-$\bar{E}$(A,f_1_1)-B(f_1_1)-e-5B(f_4_1)
A(f_1_1)-e-A^2(f_1_1)-e-C_1(A,f_1_1)-e-B(f_3_1)-4B(f_4_1)
A(f_2_1)-e-D_2(A,f_1_1)-e-3B(f_1_1)-4B(f_4_1)
F(A,f_1_1)-e-E(D,f_1_1)-E(C,f_4_1)
A(f_3_1)-A(f_1_1)-7e-D_1(A,f_1_1)-D_2(C,f_1_1)-e-$\bar{E}$(C,f_1_1)-5e-4B(f_3_1)
A^2(f_1_1)-5e-F(A,f_1_1)-e-B(f_1_1)-e-4B(f_1_1)
D_1(C,f_3_1)-e-C_3(A,f_2_1)-F(A,f_1_1)-2e-2B(f_1_1)
A(f_2_1)-8e-F(A,f_1_1)-2e-2B(f_1_1)-$\bar{B}$(B,f_1_1)-B(f_1_1)
D_1(A,f_3_1)-D_1(A,f_1_1)-e-F(D,f_1_1)-e-2B(f_1_1)

glider gun

A(f_1_1)-3e-D_1(C,f_1_1)-e-2B(f_1_1)-e-$\bar{B}$(f_1_1)
D_1(C,f_3_1)-e-C_1(A,f_1_1)-e-$\bar{E}$(B,f_1_1)
A^5(f_2_1)-3e-A(f_1_1)-3e-F(E,f_1_1)-2e-B^4(f_4_1)

$A(f_3\_1)$-8e-$C_2(A,f_3\_1)$-$C_3(A,f_1\_1)$-e-$F(A,f_1\_1)$-2e-3$B(f_1_1)$-e-4$B(f_1_1)$
$A^5(f_1\_1)$-8e-$\bar{E}(A,f_1\_1)$-e-4$B(f_1_1)$
$A^5(f_2\_1)$-6e-$\bar{E}(A,f_2\_1)$-e-4$B(f_1_1)$
$A(f_1\_1)$-7e-$\bar{E}(A,f_2\_1)$-e-$B(f_1\_1)$-$B(f_4\_1)$-3$B(f_4_1)$-e-$\bar{B}(C,f_4\_1)$-$B(f_4\_1)$

glider gun^n to $n = 1$
$F(B,f_1\_1)$-e-$E(A,f_1\_1)$-$B(f_1\_1)$-3e-$B(f_1\_1)$
$A^2(f_2\_1)$-e-$A(f_3\_1)$-e-$A(f_1\_1)$-6e-$G^4(A,f_1\_1)$

A.2 Table 2

Relations of productions to generate groups or packages of gliders.

$\bar{B}$ gliders
$A^5(f_1\_1)$-2e-$\bar{E}(B,f_1\_1)$-$\bar{E}(A,f_2\_1)$

C glider
$A(f_1\_1)$-2e-$\hat{B}(A,f_1\_1)$

D glider
A(f_1_1)-$A(f_2\_1)$-3e-$G(B2,f_2\_1)$

E and $\bar{E}$ gliders
$A(f_1\_1)$-2e-$B(C,f_1\_1)$
$D_1(C,f_2\_1)$-e-$C_3(A,f_2\_1)$-$F(A.f_1\_1)$-2e-2$B(f_1_1)$
$A(f_2\_1)$-3e-$D_1(A,f_1\_1)$-2e-$E^2(B,f_1\_1)$

F gliders
$D_1(A,f_1\_1)$-$D_1(C,f_1\_1)$-2e-$\bar{B}(A,f_2\_1)$

G gliders
$D_2(A,f_1\_1)$-e-$F(C,f_1\_1)$

A.3 Table 3

Relations of productions to generate $\bar{B}^2$ and $\hat{B}^2$ gliders.

$\bar{B}^2$ glider
$D_1(A,f_4\_1))$-e-$C_1(A,f_1\_1))$-e-$\bar{E}(A,f_1\_1))$-e-$B(f_2\_1)$-2e-3$B(f_1_1)$
2$A(f_1_1)$-8e-$C_3(B,f_2\_1)$-$F(F,f_2\_1)$-$F(G,f_1\_1)$-2e-4$B(f_4_1)$-$\bar{B}(A,f_2_1)$-e-8$B(f_1_1)$
3$A(f_1_1)$-12e-$C_1(A,f_1\_1)$-3e-$C_1(A,f_1\_1)$-$C_1(B,f_1\_1)$-$B(f_1\_1)$-e-4$B(f_1_1)$-4e-2$B(f_1_1)$
$A(f_2\_1)$-$A^2(f_2\_1)$-$A^2(f_1\_1)$-3e-$C_2(A,f_1\_1)$-8$B(A,f_1_1)$-3e-4$B(f_1_1)$

$A(\mathrm{f}_1\_1)$-e-$A(\mathrm{f}_1\_1)$-9e-$C_2(\mathrm{A},\mathrm{f}_1\_1)$-$C_1(\mathrm{A},\mathrm{f}_4\_1)$-$C_1(\mathrm{A},\mathrm{f}_1\_1)$-$e$-$\overline{B}(\mathrm{A},\mathrm{f}_1\_1)-2e$-$B(\mathrm{f}_3\_1)-3B(\mathrm{f}_1_1)$

$\hat{B}^2$ glider

3$A(\mathrm{f}_1_1)$-10e-$C_1(\mathrm{B},\mathrm{f}_3\_1)$-$e$-$C_2(\mathrm{A},\mathrm{f}_3\_1)$-$C_2(\mathrm{B},\mathrm{f}_1\_1)$-$\overline{E}(\mathrm{B},\mathrm{f}_2\_1)$-$F(\mathrm{B},\mathrm{f}_3\_1)$-$e$-$\overline{E}(\mathrm{B},\mathrm{f}_1\_1)-7e-3B(\mathrm{f}_1_1)$

$\overline{B}^{n-1}$ and $\hat{B}^{n-1}$ gliders for $n > 1$

{3e-$D_2(\mathrm{C},\mathrm{f}_1\_1)$}*-$e$-$\overline{B}^n(\mathrm{A},\mathrm{f}_1\_1)$

{3e-$D_2(\mathrm{C},\mathrm{f}_1\_1)$}*-$e$-$\hat{B}^n(\mathrm{A},\mathrm{f}_1\_1)$