

Modeling Prey-Predator Dynamics via Particle Swarm Optimization and Cellular Automata

Mario Martínez-Molina¹, Marco A. Moreno-Armendáriz¹,
Nareli Cruz-Cortés¹, and Juan Carlos Seck Tuoh Mora²

¹ Centro de Investigación en Computación,
Instituto Politécnico Nacional,
Av. Juan de Dios Bátiz s/n, México D.F., 07738, México
mariomartinezmolina@live.com

² Centro de Investigación Avanzada en Ingeniería Industrial,
Universidad Autónoma del Estado de Hidalgo,
Carr. Pachuca-Tulancingo Km. 4.5, Pachuca Hidalgo 42184, México

Abstract. Through the years several methods have been used to model organisms movement within an ecosystem modelled with cellular automata, from simple algorithms that change cells state according to some pre-defined heuristic, to diffusion algorithms based on the one dimensional Navier - Stokes equation or lattice gases. In this work we show a novel idea since the predator dynamics evolve through Particle Swarm Optimization.

1 Introduction

Cellular Automata (CA) based models in ecology are abundant due to their capacity to describe in great detail the spatial distribution of species in an ecosystem. In [4], the spatial dynamics of a host-parasitoid system are studied. In this work, a fraction of hosts and parasites move to colonize the eight nearest neighbors of their origin cell, the different types of spatial dynamics that are observed depend on the fraction of hosts and parasitoid that disperse in each generation. Low rates of host dispersal lead to chaotic patterns. If the rate of host dispersal is too low, and parasitoid dispersal rates are very high, “crystal lattice” patterns may occur. Mid to high rates of host dispersal lead to spiral patterns.

In [9], an individual-oriented model is used to study the importance of prey and predator mobility relative to an ecosystem’s stability. Antal and Droz [1] used a two-dimensional square lattice model to study oscillations in prey and predator populations, and their relation to the size of an ecosystem. Of course, organisms have multiple reasons to move from one zone of their habitat to another, whether to escape from predation, or to search the necessary resources for survival. An example appears in [8], where predators migrate via lattice gas interactions in order to complete their development to adulthood.

In this work we show a CA model of a theoretical population, where predator dynamics evolve through Particle Swarm Optimization (PSO). Each

season, predators search the best position in the lattice according to their own experience and the collective knowledge of the swarm, using a fitness function that assigns a quality level according to local prey density in each site of the lattice. To the best of our knowledge, such approach has never been used to model predator dynamics in an spatial model. The results show oscillations typical of Lotka -Volterra systems, where for each increase in the size of the population of predators, there is a decrease in the size of the population of preys.

2 Background

2.1 Cellular Automata

CA are dynamical systems, discrete in time and space. They are adequate to model systems that can be described in terms of a massive collection of objects, known as cells, which interact locally and synchronously. The cells are located on the d -dimensional euclidean lattice $L \subseteq \mathbb{Z}^d$. The set of allowed states for each cell is denoted by Q . Each cell changes its state synchronously at discrete time steps according to a local transition function $f : Q^m \rightarrow Q$, where m is the size of the d -dimensional neighborhood vector N defined as:

$$N = (\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3, \dots, \mathbf{n}_m) \quad (1)$$

where $\mathbf{n}_i \in \mathbb{Z}^d$. Each \mathbf{n}_i specifies the relative locations of the neighbors of each cell [6], in particular, cell \mathbf{n} has coordinates $(0, 0, \dots, 0)$ and neighbors $\mathbf{n} + \mathbf{n}_i$ for $i = 1, 2, \dots, m$. A configuration of a d - dimensional cellular automaton is a function:

$$c : \mathbb{Z}^d \rightarrow Q$$

that assigns a state to each cell. The state of cell $\mathbf{n} \in \mathbb{Z}^d$ at time t is given by $c^t(\mathbf{n})$, the set of all configurations is $Q^{\mathbb{Z}^d}$. The local transition function provokes a global change in the configuration of the automata. Configuration c is changed into configuration c' , where for all $\mathbf{n} \in \mathbb{Z}^d$:

$$c'(\mathbf{n}) = f [c(\mathbf{n} + \mathbf{n}_1), c(\mathbf{n} + \mathbf{n}_2), \dots, c(\mathbf{n} + \mathbf{n}_m)] \quad (2)$$

The transformation $c \mapsto c'$ is the global transition function of the cellular automaton, defined as:

$$G : Q^{\mathbb{Z}^d} \rightarrow Q^{\mathbb{Z}^d} \quad (3)$$

In a two dimensional cellular automaton the Moore neighborhood is often used, such neighborhood can be generalized as the d -dimensional M_r^d neighborhood [6] defined as:

$$(\mathbf{n}_{i_1}, \mathbf{n}_{i_2}, \dots, \mathbf{n}_{i_d}) \in \mathbb{Z}^d \text{ where } |n_{i_j}| \leq r \text{ for all } j = 1, 2, \dots, d \quad (4)$$

2.2 Particle Swarm Optimization

Particle Swarm Optimization is a bio-inspired algorithm based on the collective behavior of several groups of animals (flocks, fish schools, insect swarms, etc) [5]. The objective of PSO is the efficient exploration of a solution space, each individual in a 'community' is conceptualized as a particle moving in the hyperspace. Such particles have the capacity to 'remember' the best position they have been in the solution space, furthermore in the global version of PSO, the best position found thus far is known to every particle of the swarm.

The position X_i^t of every particle in the swarm is updated in discrete time steps according to the next equations:

$$V_i^{t+1} = \omega V_i^t + k_1 r_1 (P_i^t - X_i^t) + k_2 r_2 (P_g^t - X_i^t) \tag{5}$$

$$X_i^{t+1} = X_i^t + V_i^{t+1} \tag{6}$$

where V_i^t is the velocity vector at time t associated to particle i , the constants k_1 and k_2 determine the balance between the experience of each individual (the cognitive component) and the collective knowledge of the swarm (the social component) respectively [2]. $r_1 \in [0, 1]$ and $r_2 \in [0, 1]$ are random variables with a uniform distribution. The best position found by the particle i is denoted by P_i , similarly the best position found by the swarm is denoted by P_g . The term ω is known as inertia weight and serves as a control mechanism to favor exploration of the solution space or exploitation of known good solutions. In [7] it is suggested to start the algorithm with $\omega = 0.9$ and linearly decrement it to $\omega = 0.4$, thus at the beginning of the algorithm exploration is favoured, and at the end exploitation is enhanced. Figure 1 shows the position updating scheme according to equations 5 and 6.

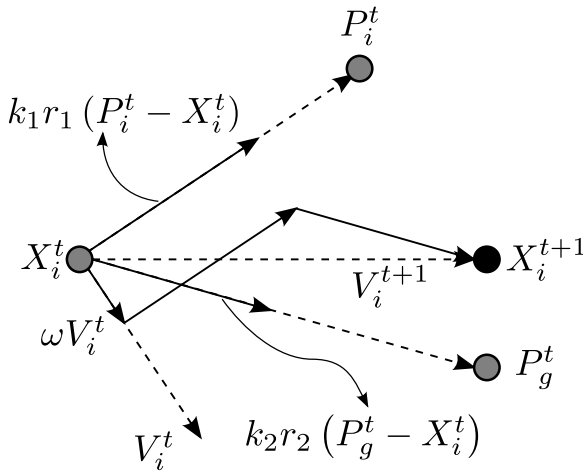


Fig. 1. Position updating scheme in PSO [11]

3 Proposed Model

Our model describes a theoretical ecosystem, where a sessile prey and a predator live. The individuals of the prey species compete locally with other members of their own species (interspecific competence), prey reproduction is a local process. In order to secure their own future, and that of their progeny, predators migrate each season from zones low on resources (preys) to zones highly abundant in food, just as in the case of preys, predators reproduce locally.

The space in which species live and interact is represented by the lattice $L \subset \mathbb{Z}^2$, periodic boundaries have been implemented, i.e. the cellular space takes the form of a torus. The set of allowed states for each cell is:

$$Q = \{0, 1, 2, 3\} \quad (7)$$

where:

- 0 is an empty cell.
- 1 is a cell inhabited by a prey.
- 2 is a cell inhabited by a predator.
- 3 is a cell containing a prey and a predator at the same time.

Both preys and predators, obey a life cycle that describes their dynamics in a generation. Predator dynamics are modelled through the next rules:

1. **Migration.** During this stage, predators move within the cellular space according to their own experience and the collective knowledge of the swarm.
2. **Reproduction.** Once the migration is complete, each predator produces new individuals at random inside a Moore neighborhood of radius two.
3. **Death.** Predators in cells lacking a prey die by starvation.
4. **Predation.** Preys sharing a cell with a predator die due to predator action.

On the other hand, the life cycle of preys is modelled under the following assumptions:

1. Intraspecific competence. Preys die with a probability proportional to the number of individuals of the prey species surrounding them, this rule uses a Moore neighborhood of radius 1. If $c^t(\mathbf{n}) = 1$, then the probability of death ($c^{t+1}(\mathbf{n}) = 0$) is given by:

$$\rho(\text{death}) = \frac{\alpha x}{m} \quad (8)$$

where:

- $\alpha \in [0, 1]$ is the intraspecific competence factor, which determines the intensity of competence exercised by preys in the neighborhood of cell \mathbf{n} .
 - x is the number of preys in the neighborhood of cell \mathbf{n} .
 - $m = |N|$.
2. Reproduction. Like predators, preys spawn new individuals at random in a Moore neighborhood of radius 2.

Each stage in preys and predators dynamics occurs sequentially. They form a cycle that defines one generation in their life, such cycle is:

1. Intraspecific Competence of preys.
2. Migration of predators.
3. Predators reproduction.
4. Predators death.
5. Predation
6. Prey reproduction.

As this cycle suggests, at each stage the rule applied to cells changes accordingly.

4 PSO as a Migration Algorithm

The main contribution in this work is to utilize a PSO algorithm as a mechanism to model the migration of predators, that is, predators change their position according to PSO. Some important differences in the use of PSO as a migration algorithm and its use in numerical optimization are:

- **Fitness.** In numerical optimization, it is common to use the same function to optimize as a mean to obtain a measure of a solution's fitness. In the proposed model, the solution space is the lattice of the CA, so each cell represents a candidate solution to the problem of finding the necessary resources for survival and procreation. Since the landscape of an ecosystem changes continuously, it is impossible to speak of an absolute best cell, instead each predator moves to the known "good" enough zones and exploits them. Once depleted, predators migrate to search for new zones for feeding and procreation, so instead of aiming for a global optima, predators exploit known local optima.
- **Solution space.** As stated before, the lattice takes the form of a torus and represents the solution space in which each particle of the swarm moves. Thus the movement of a particle can take a predator from one edge of the lattice to the other, this favours exploration.
- **Swarm size.** In our model each particle is also a predator, in consequence, they can die, and they can reproduce, this changes the size of the swarm in each generation.

Since the model is discrete in space, the update of a particle's position simply determines the location to which the particle moves to. Consequently the cell from which the predator initiates its migration could go through the following state changes:

$$c^t(\mathbf{n}) = 2 \rightarrow c^{t+1}(\mathbf{n}) = 0$$

$$c^t(\mathbf{n}) = 3 \rightarrow c^{t+1}(\mathbf{n}) = 1$$

Similarly, the cell in which the predator ends its migration could experience the next state transitions:

$$\begin{aligned}c^t(\mathbf{n}) = 0 &\rightarrow c^{t+1}(\mathbf{n}) = 2 \\c^t(\mathbf{n}) = 1 &\rightarrow c^{t+1}(\mathbf{n}) = 3\end{aligned}$$

As a measure of a particle's fitness, we use prey density in the neighborhood N of each cell, thus, a cell with more preys in its neighborhood is a better location than a cell with less preys in its neighborhood.

4.1 Migration Process

As stated in section 3, migration takes place after the competence of preys. At the beginning of each migration, particles determine the fitness of their current position (by measuring prey density in its neighborhood), and set their best known position. Using this information, the best known position of the swarm is set. After this initialization step, migration proceeds as follows:

1. The velocity vector of each particle is updated according to equation 5, the magnitude of which depends on the values taken by parameters ω , k_1 , k_2 , r_1 and r_2 .
2. Each particle moves to its new position by adding the vector V_i^{t+1} to its current position X_i^t .
3. The new neighborhood is explored and if necessary, both the best known position of each particle P_i^t , and the best position of the swarm (P_g^t) are updated.
4. The value of the inertia weight ω is adjusted.

This process is repeated 5 times, to ensure a good search in the proximity of the zones known by the swarm and by each individual particle. Figure 2 shows the migration of a swarm of 3 particles through PSO. The states of each cell are shown with the next color code:

- **Black:** empty cell - state 0
- **Light gray:** prey - state 1
- **Dark gray:** predator - state 2
- **White:** cell inhabited by a prey and a predator at the same time.

Figure 2a shows initial conditions, of the 3 individuals, the one located at the bottom - right is the one with the best fitness, so the other two will move in that direction (Figures 2b and 2c). When predators end their migration, they reproduce, so by migrating to zones with a high prey density, not only they have a better chance of survival, but their offspring too.

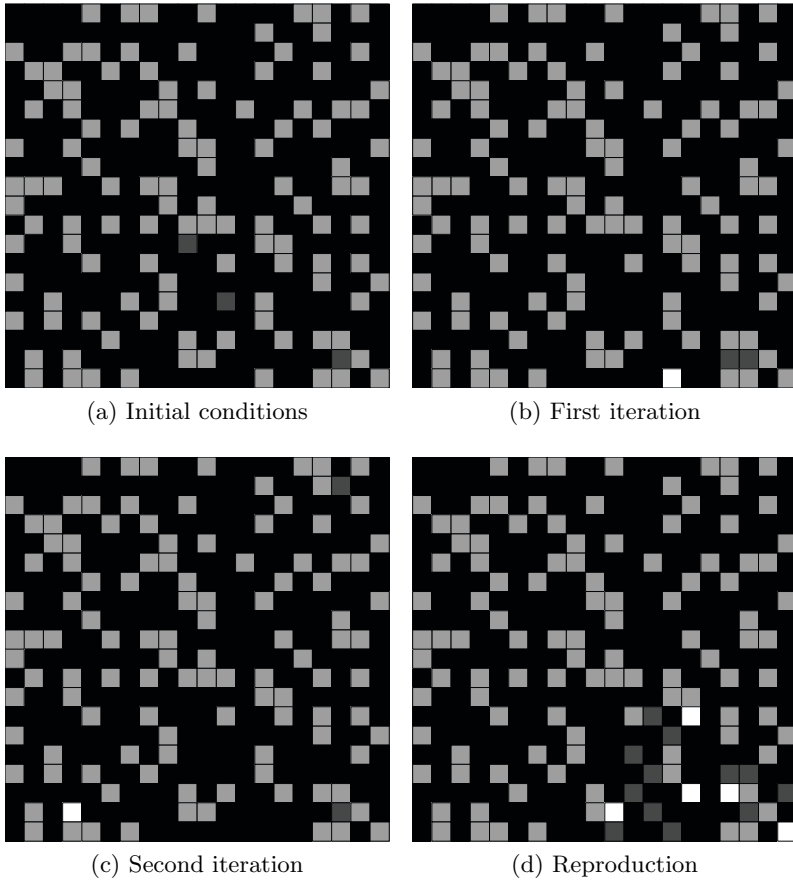


Fig. 2. Migration through PSO

5 Comparison with Lotka - Volterra Systems

The growth of a population in the absence of predators and without the effects of intraspecific competence can be modeled through the differential equation [3]:

$$\frac{dZ}{dt} = \gamma Z \quad (9)$$

where:

- Z is the size of the population.
- γ is the population's rate of growth.

However, when predation is taken into account, the size of the population is affected proportionally to the number of predator-prey encounters, which depend on the size of the populations of preys (Z) and predators (Y). Since predators

are not perfect consumers, the actual number of dead preys depends on the efficiency of the predator. Let a be the rate at which predators attack preys, thus the rate of consumption is proportional to aZY , and the growth of the population is given by:

$$\frac{dZ}{dt} = \gamma Z - aZY \tag{10}$$

Equation 10 is known as the Lotka-Volterra prey equation. In the absence of preys, the population of predators decay exponentially according to:

$$\frac{dY}{dt} = -sY \tag{11}$$

where s is the predator mortality rate. This is counteracted by predator birth, the rate of which depend on only two things: the rate at which food is consumed, aZY , and the predator's efficiency h , predator birth rate is $haZY$, thus:

$$\frac{dY}{dt} = haZY - sY \tag{12}$$

Equation 12 is known as the Lotka-Volterra predator equation. Figure 3 shows the dynamics of an ecosystem ruled by equations 10 and 12.

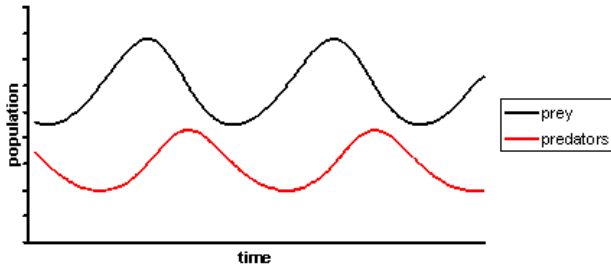


Fig. 3. Lotka - Volterra prey - predator dynamics

The Lotka-Volterra equations show periodic oscillations in predator and prey populations. This is understandable given the next reasoning: when there is an abundant number of preys, the food consumption by predators increases, and thus the number of predators grows. Due to this fact, the number of prey diminishes, and so does the food available to predators, which increase predator mortality. The death of predators allows a new increase in the population of preys, and the process begins anew. An excellent review of lattice based models that give new perspectives on the study of oscillatory behavior in natural populations appears in [10].

It is possible to simulate the behavior of Lotka-Volterra equations through the proposed model, most of the parameters of these equations are indirectly taken into account in such model, e. g., predator efficiency depends on whether predators have a successful migration or not. To simulate the behavior of equations 10 and 12, the next parameters are used.

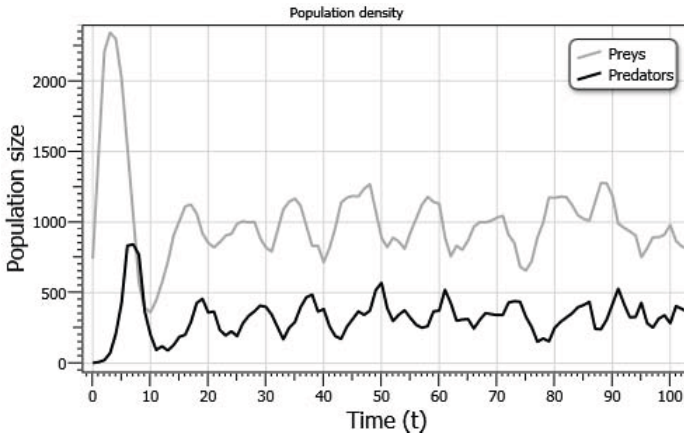


Fig. 4. Prey - predator dynamics through PSO in a CA

- Size of the lattice: $50 \times 50 = 2500$ cells.
- Initial prey percentage: 30%
- Intraspecific competence factor: $\alpha = 0.3$. If this parameter is too high, most of the ecosystem will be composed of small “patches” of preys separated by void zones, in consequence only a fraction of predators will survive the migration.
- Mean offspring of each prey: 3 individuals.
- Swarm’s initial size: 3 particles.
- Mean offspring of each predator: 5 individuals. A high predator reproductive rate would lead to over-exploitation of resources, in consequence there is a chance that predators will go extinct.
- $k_1 = 2.0$ and $k_2 = 2.0$.
- Initial inertia weight $\omega = 0.9$ and Final inertia weight $\omega = 0.4$
- $|V_{max}| = \frac{\text{lattice width}}{3}$

Figure 4 shows the dynamics of the proposed model, oscillations obeying the abundance cycles of prey and predators are shown. Figure 5a shows a swarm about to begin a migration, after feeding on preys (Figure 5b), there is a wide empty zone where most of the cells have a fitness equal to zero. In order to survive, predators move to “better” zones. In Figure 5c most of the swarm has moved away from the empty zone (differences in the distribution of prey are due to the process of competence and reproduction of the past iteration) to zones with a higher density of preys. The migration of predators allows the colonization of the previously predated zone, meanwhile recently attacked zones will be reflected in a decrease in the population of preys (Figure 5d).

5.1 Extinction

A small population of predators with a high reproductive capacity might lead to over-exploitation of resources (Figure 6a). Figure 6d shows the results of a

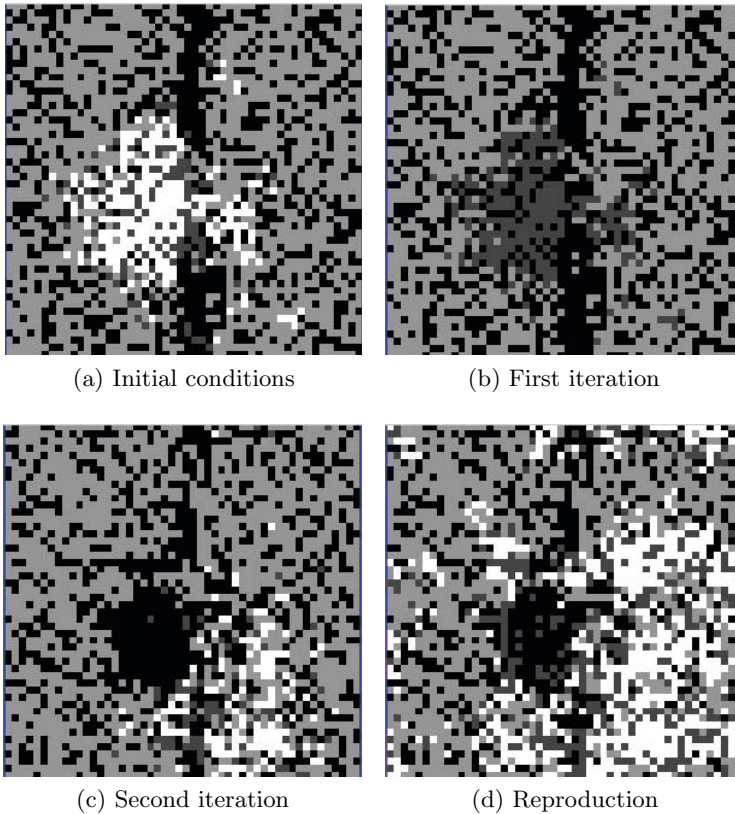
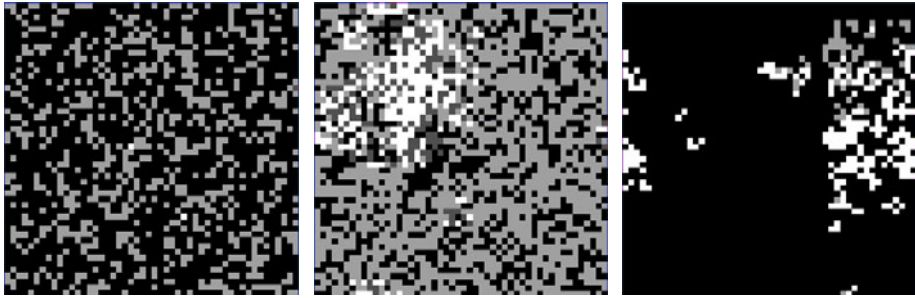


Fig. 5. Spatial dynamics in the proposed model

simulation where each predator has a mean offspring of 15 individuals. As the size of the swarm grows (Figure 6b), bigger patches of preys are destroyed, and eventually migration becomes too difficult for most of the predators (Figure 6c). Each passing generation, the number of surviving predators decreases, until the whole population becomes extinct.

5.2 Discussion

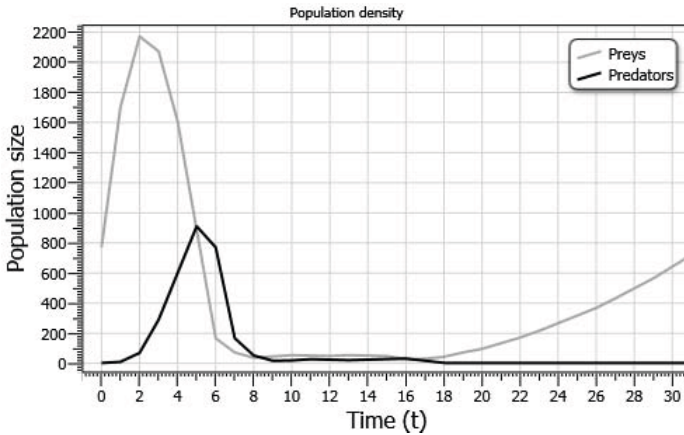
There are other experiments that are worth discussing. It is possible to adjust the range of local search by altering the value of the inertia weight ω . By setting “high” initial and final values for this parameter, it is possible to increase the radius of local search, particles explore a wider area in the vicinity of known good zones. In consequence, most particles become disperse, and if resources are abundant, a higher predation efficiency is achieved; but if resources are sparse, the search will lead them to zones devoid of resources, and most of them will die. On the other hand, “smaller” values for the inertia weight will produce a very compact swarm specialized in local exploitation of resources.



(a) Initial conditions

(b) Population growth

(c) Over-exploitation



(d) Extinction dynamics

Fig. 6. Predators extinction

It is necessary to determine the relation between the size of the lattice, and the long term dynamics of the model. Other works [12] [1], have reported oscillations of the Lotka-Volterra type only when the size of an ecosystem is “large enough”.

6 Conclusions and Future Work

We have presented a CA based model of a theoretical ecosystem where predators migrate through PSO in order to find resources. Here we have presented the simplest implementation of PSO, yet the results are promising, it is certainly possible to establish some other fitness measures, thus it would be possible for organisms to move according to some other factors, i.e. temperature, pollution, chemical factors, etc. Of course, it is necessary to analyse the full dynamics of the model, in order to establish its strengths and weaknesses. A substantial improvement of the model would be the implementation of the local PSO, this will allow individuals to react to the information received from local members of the swarm in a finite neighborhood, thus allowing a more realistic modeling, where individuals only have access to the information of their nearest neighbors.

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References

1. Antal, T., Droz, M.: Phase transitions and oscillations in a lattice prey-predator model. *Physical Review E* 63 (2001)
2. Banks, A., Vincent, J., Anyakoha, C.: A review of particle swarm optimization Part I: background and development. *Natural Computing* 6(4) (2007)
3. Begon, M., Townsend, C.R., Harper, J.L.: *Ecology: From Individuals to Ecosystems*, 4th edn. Blackwell Publishing (2006)
4. Comins, H.N., Hassell, M.P., May, R.M.: The spatial dynamics of host-parasitoid systems. *The Journal of Animal Ecology* 61(3), 735–748 (1992)
5. Eberhart, R., Kennedy, J.: A new optimizer using particle swarm theory. In: *Proceedings of the Sixth International Symposium on Micro Machine and Human Science*, pp. 39–43 (1995)
6. Kari, J.: Theory of cellular automata: a survey. *Theoretical Computer Science* 334, 3–33 (2005)
7. Kennedy, J., Eberhart, R.C., Shi, Y.: *Swarm Intelligence*, 1st edn. Morgan Kaufman (2001)
8. van der Laan, J.D., Lhotka, L., Hogeweg, P.: Sequential predation: A multi-model study. *Journal of Theoretical Biology* 174, 149–167 (1995)
9. Mccauley, E., Wilson, W.G., de Roos, A.M.: Dynamics of age-structured and spatially structured predator-prey interactions: Individual-based models and population-level formulations. *American Naturalist* 142(3), 412–442 (1993)
10. Pekalski, A.: A short guide to predator-prey lattice models. *Computing in Science and Engineering* 6(1) (2004)
11. Shi, Y., Liu, H., Gao, L., Zhang, G.: Cellular particle swarm optimization. In: *Information Sciences - ISCI (2010)*
12. Wolff, W.F.: Microinteractive predator-prey simulations. *Ecodynamics: Contributions to Theoretical Ecology* pp. 285–308 (1988)